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Westinghouse

EC 232

ATTENUATION OF BEARING TRANSMITTED NOISE

Volume 3

October 1964

performed in conjunction with

subcontractor

Mechanical Technology Incorporated

in fulfillment of

Contract No. NOBS-86914

Bureau of Ships

Department of Navy

U. S. of America

Westinghouse Electric Corporation
Lester, Pennsylvania

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PREFACE

This study covers the analysis and experimental investigation of the effect of a hydraulically-supported, pivoted-shoe, journal bearing on the attenuation of noise originating from rotor unbalance.

This work is in fulfillment of Bureau of Ships Contract No. NObs-86914 and it has been assigned Project Serial No. SF 013-11-05, Task 3679.

The final report includes three volumes, Volume 3 being herewith included.

Volume 1 - Spring and Damping Coefficients for the Tilting-Pad Jounral Bearing.

Volume 2 -

Part I: Attenuation of Rotor Unbalance Forces by Flexible Bearing Supports

Part II: Unbalance Response of a Uniform Elastic Rotor Supported in Damped Flexible Bearings

Volume 3 -

Part I: A General Computer Program for Unbalance Response of a Rotor in Fluid Film Journal Bearings

Part II: Experimental Investigation of Hydraulic-Supports

Mechanical Technology Incorporated was primarily responsible for the analytical portion of this study, while Westinghouse Electric Corporation designed and conducted the experimental test.

Volume 3

Part I

A General Computer Program for Unbalance Response of a Rotor in Fluid Film Jounral Bearings

by

Jorgen Lund

Mechanical Technology Incorporated

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INTRODUCTION

A rotor supported in fluid film journal bearings is a complex dynamical system and exhibits a variety of characteristics: critical speeds, instabilities, unbalance vibrations, etc. In many applications it is a critical member and any design procedure requires, as a minimum, the calculation of the critical speeds of the rotor. Other investigations may include a check on the stability of the rotor (oil whip, fractional frequency whirl) or calculating the rotor amplitude caused by an external excitation (e.g. shock loading). In the present case the concern is with the response of the rotor to unbalance forces and to determine both the whirl amplitudes and the forces transmitted to the foundation.

The fluid film journal bearings play a very important role in the dynamics of the rotor. They are normally the predominant source of damping such that without this source it would be impossible to run the rotor through any of its critical speeds. Secondly, the bearing film is flexible and thereby it may lower the critical speeds drastically (e.g. for the first critical speed the reduction can be 30 to 40 per cent, or even more). The fluid film flexibility also causes the bearing to act as a vibration isolator, attenuating the dynamical forces transmitted to the pedestals. Hence, in any comprehensive rotor response calculation it is necessary to have a method available which treats the dynamical bearing forces as accurately as possible. It is the purpose of this report to present such a calculation method and to describe a computer program for carrying out the numerical computations.

The computer program is very general. It calculates the rotor whirl amplitude and the force transmitted to the base due to a given rotor unbalance. The rotor is flexible and may have any arbitrary geometry. Also, there can be splined couplings in the rotor and several bearings. The bearing pedestals can be assigned both flexibility and damping. Since the bearing film forces are not the same in all directions the whirl motion of the rotor is treated as two-dimensional such that it becomes an orbit around the equilibrium position. The orbit is elliptical and its dimensions and orientation vary along the length of the rotor. The computer program calculates the whirl orbits for a number of points along the rotor and gives also the components of the force transmitted to the foundations.

The report sets forth the analysis for performing the calculations and describes the computer program based on the analysis. Detailed instructions for preparing the computer input and interpretating the output are given.

DISCUSSION

a. General

The rotor analysis derives in its principle from the Myklestad-Prohl method (Ref. 1,2,3). However, in its original form the Myklestad-Prohl method is set up only for calculating the critical speeds of flexible rotor in flexible bearings and it treats the rotor motion as a transverse vibration of a beam. In the present case the motion is treated as two-dimensional, damping is included in the bearings in addition to stiffness and the rotor response is calculated at any speed, not just the mode shape at the critical speeds. Furthermore, the effect of gyroscopic moments is included.

In general a rotor's cross-sectional dimensions and its mass distribution varies along the length of the rotor. Thus, for calculation purposes it is convenient to break the rotor up into short sections, each section having a constant cross-section. Furthermore, when there are many sections the mass of each section can be divided into two parts and lumped at the end points of the section. Concentrated masses like wheels, impellers, etc. can be made to coincide with an end point of a section. In this way the rotor is replaced by an idealized model consisting of a number of mass points connected by weightless, flexible bars. The model can be brought as close to the actual rotor as desired by making the subdivisions small but in practice only a limited number of divisions is needed to obtain a very good accuracy.

Since the bearing film properties to a large extent control the whirl motion of the rotor, it is necessary to represent the dynamical bearing film forces as accurately as possible. The method of representation is based on the assumption that the whirl amplitude is small compared to the bearing clearance such that the dynamical forces can be replaced by their gradients around the steady state journal center position. In this way the dynamical forces become proportional to the whirl amplitude and the corresponding velocity, and the factors of proportionality are called spring and damping coefficients. They differ from conventional mechanical spring-dashpot systems by also containing cross-coupling terms in addition to direct-coupling terms, i.e. the dynamical force in a given direction (say the x-direction) is not only proportional to the amplitude and velocity components in that direction but is also proportional to the amplitude and velocity components in the mutually perpendicular direction (i.e. the y-direction). Hence, in an arbitrary reference coordinate system with x and y-axis the two dynamical force components can be expressed by:

$$F_x = -K_{xx}x - C_{xx}\dot{x} - K_{xy}y - C_{xy}\dot{y}$$
$$F_y = -K_{yx}x - C_{yx}\dot{x} - K_{yy}y - C_{yy}\dot{y}$$

where x and y are the amplitude components, $\dot{x}$ and $\dot{y}$ are the velocity components, K_{xx} and K_{yy} are the direct coupling spring coefficients, C_{xx} and C_{yy} are the direct-coupling damping coefficients, K_{xy} and K_{yx} are the cross-coupling spring coefficients, and C_{xy} and C_{yx} are the cross-coupling damping coefficients. These 8 coefficients are functions of the bearing Sommerfeld number defined through the rotor speed, the steady state bearing reaction, the lubricant viscosity and the bearing dimensions (for gas bearings the coefficients are functions of the compressibility number and the bearing eccentricity ratio). Thus, the coefficients vary with speed. A method for calculating the coefficients is given in Refs. 4 and 9 and values of the coefficients for several bearing types may be found in Refs. 4,5,6,7 and 8.

Frequently the pedestals, on which the bearings are mounted, are as flexible as the bearing film. In such cases the pedestal stiffness must be included in the calculations. For completeness the analysis allows for both stiffness, damping and inertia in the pedestals. Furthermore, as the rotor bends under the influence of the unbalance forces the journals become cocked in their bearings. The fluid film resist the tilting and this can be expressed by a set of 8 spring and damping coefficients in analogy to the previously discussed coefficients. The analysis includes this effect, both in the bearings and in the pedestals. The resistance to tilt normally affects the rotor motion only at speeds above the second or third critical speed but if the pedestals are made soft for alignment purposes resonance conditions may exist which can only be explored if the effect of tilt is included.

Occasionally the rotor is not a single member but consist of several rotors connected by splined couplings (e.g. a turbine-generator set connected by a splined coupling). The analysis allows for including splined couplings anywhere in the rotor and assumes that no bending moment is transfered through the coupling.

The whirling motion of the rotor is generated by unbalances built into the rotor. In general the unbalance varies in magnitude and circumferential location along the rotor such that under speed the unbala ce forces may bend the rotor into complicated shapes (e.g. resembling a "cork-screw"). The bend rotor whirls around its steady state position (i.e. the position the rotor would occupy if there were no unbalance forces)with each point of the rotor axis describing an elliptical path. The dimensions and orientation of the ellipse varies along the length of the rotor.

If the rotor runs at high speed and has large discs (e.g. turbine wheels, etc) mounted on the shaft the gyroscopic moment becomes important, especially if a wheel is overhung at one end of the rotor. The gyroscopic moment is proportional to the mass-moment of inertia of the wheel, the square of the speed and the deflection angle of the rotor. If the rotor motion is considered as a transverse vibration of a beam (i.e. the whirl orbit is a straight line) the gyroscopic moment tends to "soften" the rotor and lower the critical speed. On the other hand, if the bearing spring and damping coefficients are the same in the vertical and the horizontal direction the rotor whirl orbit becomes a circle and the gyroscopic moment stiffens the rotor. Actually, the whirl orbit is elliptical, i.e. somewhere between a straight line and a circle, and the effect of the gyroscopic moment can only be assessed by performing the complete rotor analysis. It is a non-linear effect since it depends on the dimensions of the elliptical whirl orbit. In the present analysis the gyroscopic moment is taken into account and is calculated by an iteration procedure.

b. <u>Special Considerations in Performing the Numerical Calculations</u>

The greatest difficulty encountered in performing the numerical calculations is the magnitude of the numbers and the loss of significant figures. These difficulties become pronounced when: a) there is an excessive number of rotor mass stations, b) the rotor is very stiff and, c) the bearings are very stiff. There is no universal remedy for the problem but if trouble arises two possibilities may be tried: a) reduce the number of rotor stations to the essential minimum and, b) apply a scale factor.

Let the scale factor be α . Then:

multiply the speed by α

multiply (EI) by α^2 (e.g. multiply E by α^2)

multiply the bearing spring and damping coefficients by α^2

(i.e. multiply K_{xx}, ωC_{xx}, M_{xx}, ωD_{xx} etc. by α^2)

multiply the pedestal stiffness by α^2 and the pedestal damping coefficients by

(i.e. multiply $\mathcal{K}_x$ and $\mathcal{K}_y$ by α^2, σ_x and σ_y by α)

leave the rotor masses, the rotor length, the pedestal masses and the unbalance unchanged.

The the numerical results will give the amplitude unchanged whereas the bending moment and the transmitted force must be divided by α^2 to obtain the actual values.

c. <u>Analysis and Dimensionless Equations</u>

Referring to the sign convention given in Fig. 2 and considering first a continuous rotor the three basic equations for determining the rotor motion are:

(1-a) Force balance for a shaft increment, dz : $\frac{dV}{dz} = \rho A \omega^2 (x+e)$

(2-a) Moment balance for a shaft increment, dz : $\frac{dM}{dz} = V + \omega^2 (i_p - i_T)\frac{dx}{dz}$

(3-a) Shaft deflection : $M = EI \frac{d^2x}{dz^2}$

where:

x - amplitude in vertical direction, inch

y - amplitude in horizontal direction, inch

z - coordinate along the rotor length, inch

e - eccentricity between mass center and shaft center, inch

A - cross-sectional area of shaft, in^2

I - cross-section moment of inertia, in^4

E - Youngs modulus, lbs/in^2

ρ - mass density, $lbs.sec^2/in^4$

$(i_p - i_T)$ - mass moment of inertia per unit length, which is effective in gyroscopic moment, $lbs.sec^2$

ω - angular speed, radians/sec

M - bending moment, lbs.in

V - shear force, lbs.

These three equations may be combined to give the familiar 4th-order differential equation governing the unbalance vibrations of a rotor:

$$\frac{d^2}{dz^2}\left(EI\frac{d^2x}{dz^2}\right) = \rho A\omega^2(x+e) + \omega^2\frac{d}{dz}\left[(i_p - i_T)\frac{dx}{dz}\right] \tag{4-a}$$

(see Ref. 3, page 330)

For a circular whirl orbit:

$$(i_p - i_T) = \rho I$$

For a straight line orbit:

$$(i_p - i_T) = -\rho I$$

For an elliptical whirl orbit, see Eq.(28) and (29) in this report.

At the bearings there is an abrupt change in the shear force and the bending moment due to the bearing reactions. Let the bearing be at $z = z_o$. Then:

$$V_{z=z_o^+} - V_{z=z_o^-} = -(K_{xx} + i\omega C_{xx})x - (K_{xy} + i\omega C_{xy})y \tag{5-a}$$

$$M_{z=z_o^+} - M_{z=z_o^-} = (M_{xx} + i\omega D_{xx})\frac{dx}{dz} - (M_{xy} + i\omega D_{xy})\frac{dy}{dz} \tag{6-a}$$

where K_{xx}, C_{xx}, M_{xx}, D_{xx} etc. are the bearing spring and damping coefficients. Actually, the effect of the pedestal should be included in the above equations as shown in Eq. (12) and (13) in the analysis.

The numerical method uses Eqs. (1-a), (2-a) and (3-a) by rewriting them into finite difference form:

$$\Delta V = \omega^2 (\rho A \Delta z) \cdot (x + e)$$

$$\Delta M = V \cdot \Delta z + \omega^2 \left[(i_p - i_T) \Delta z \right] \left(\frac{dx}{dz} \right)$$

$$\Delta\left(\frac{dx}{dz}\right) = \int_z^{z+\Delta z} \frac{M}{EI}\, dz$$

$$\Delta x = \left(\frac{dx}{dz}\right)\Delta z + \int_z^{z+\Delta z}\int_z^{z+\Delta z} \frac{M}{EI}\, dz\, dz$$

Together with Eqs. (5-a) and (6-a) these equations form a set of recurrence relationships which can be solved step by step, starting from one end of the rotor until reaching the other end. The details are given later.

Occasionally it is desired to perform a dimensionless analysis. The two governing quantities are:

(7-a) $$\omega_n^2 = \frac{(EI)_0}{\ell^3 M_T} = \frac{K_r}{M_T}$$

(8-a) $$K_r = \frac{(EI)_0}{\ell^3}$$

where:

$(EI)_0$ - reference value of EI, lbs.in^2

ℓ - rotor span between bearings, inch

$M_T = \int_0^\ell \rho A\, dz$, total rotor mass, lbs.sec^2/in

K_r - rotor stiffness, lbs/in

ω_n - equal to or proportional to a critical rotor speed, radians/sec

For a uniform shaft (EI = constant, A = Constant):

$$\omega_n^2 = \frac{\pi^2 n^4}{4} \frac{EI}{\ell^3 M_T} = \frac{\left(\frac{\pi^2 n^4}{4} EI\right)}{\ell^3 M_T}$$

where n designates the order of the critical speed. Thus, for the first mode: $n=1$ i.e.,

$$(EI)_0 = \frac{\pi^2 n^4}{4} EI = 2.4674 \cdot EI \qquad \text{(Uniform shaft, first mode)}$$

However, it is not necessary that ω_n be a critical speed but Eq.(7-a) must be satisfied.

The dimensionless parameters become:

$$\begin{aligned}
x' &= x/e_0 \\
z' &= z/\ell \\
(EI)' &= EI/(EI)_0 \\
V' &= V/e_0 K_r \\
M' &= M/e_0 K_r \ell \\
K'_{xx} &= K_{xx}/K_r = \left(\frac{W_0}{C_0 K_r}\right)\left(\frac{C_0 W}{C W_0}\right)\left(\frac{C K_{xx}}{W}\right) \\
(\omega C'_{xx}) &= \omega C_{xx}/K_r = \left(\frac{W_0}{C_0 K_r}\right)\left(\frac{C_0 W}{C W_0}\right)\left(\frac{C \omega C_{xx}}{W}\right) \\
M'_{xx} &= M_{xx}/K_r \ell^2 = \left(\frac{W_0}{C_0 K_r}\right)\left(\frac{C_0 W}{C W_0}\right)\left(\frac{L}{\ell}\right)^2\left(\frac{C M_{xx}}{W L^2}\right) \\
(\rho A)' &= \ell \rho A/M_T \\
(i_p - i_T)' &= (i_p - i_T)/M_T \ell
\end{aligned}$$

where:

- e_0 - reference value for the rotor mass eccentricity, inch
- C_0 - reference value for the radial bearing clearance, inch
- C - actual radial bearing clearance, inch
- W_0 - reference value for the bearing reaction, lbs.
- W - actual bearing reaction, lbs.
- L - bearing length, inch

The dimensionless bearing coefficients are given the form above since the values obtained from lubrication theory are CK_{xx}/W, $C\omega C_{xx}/W$, etc. Normally, a dimensionless analysis is only performed for a simple system where all bearings are identical, i.e. $C=C_0$ and $W=W_0$. In that case the basic dimensionless parameters are:

speed ratio: $\left(\frac{\omega}{\omega_n}\right)$

dimen ionless ro_or stiffness: $K'_r = CK_r/W$

dimensionless bearing coefficients: CK_{xx}/W, $C\omega C_{xx}/W$, $\left(\frac{L}{\ell}\right)^2 (CM_{xx}/WL^2)$, etc.

Thus, to perform a dimensionless calculation for a given value of K_r' use as input to the computer program:

Speed $= \left(\frac{\omega}{\omega_n}\right)/.10471976$

Mass at station $i = \frac{m_i}{\sum^n m_j}$ 3.86069-10^5 (m=station weight, lbs; n=number of stations)

$(I_p - I_T)$ at station $i = \frac{(I_p - I_T)_i}{M_T \ell^2}$ 3.86069-10^5

Cross-sectional moment of inertia for section $i-(i+1)$: $1000 \cdot I/I_o$

Young modulus = 1

Length of section $i-(i+1) = \ell_i/\ell$

Bearing spring coefficient $= \frac{1}{K_r'}\left(\frac{CK_{xx}}{W}\right)$

Bearing damping coefficient $= \frac{1}{K_r'}\left(\frac{C\omega C_{xx}}{W}\right)$

Unbalance such that: $\sum_1^n U_x\,(\text{oz.in}) = 6177.1 \qquad \sum_1^n U_y\,(\text{oz.in}) = 6177.1$

Then the computer output will give:

amplitude $= \frac{x}{e_o}$ and $\frac{y}{e_o}$

bending moment $= M' = M/e_o K_r \ell = M/\frac{e_o}{C} W \ell K_r'$

transmitted force = (actual force)/$e_o K_r$ = (actual force)/$\frac{e_o}{C} W K_r'$

ANALYSIS AND DESCRIPTION OF THE COMPUTER PROGRAM

The remaining part of the report describes the basic analysis and gives the detailed instructions for using the computer program:

PN0011: "Unbalance Response of a Rotor in Fluid Film Journal Bearings" for the IBM 704 digital computer. This program calculates the rotor deflection and bending moment, the pedestal deflection and the transmitted force resulting from a specified rotor unbalance. It differs from conventional programs by taking into account the variation of support flexibility and damping along the whirl path of the rotor.

The supports for the rotor consist of a fluid film bearing on a pedestal, both members possessing flexibility and damping for translatory and rotational motion. The flexibility and damping are linear in displacement and velocity respectively, the proportionality factors denoted as spring and damping coefficients. The fluid film is represented by 4 spring coefficients and 4 damping coefficients for translatory motion and similarly for rotational motion, thus allowing for coupling between the motion in two mutually perpendicular directions. The pedestal has no such coupling and is represented by 2 spring and 2 damping coefficients for both translatory and rotational motion with corresponding pedestal mass and mass moment of inertia. Hence, each point of the rotor will whirl in an elliptic path around its steady state position.

In addition, the program includes the effect of gyroscopic moment and provides for couplings in the rotor.

THEORETICAL ANALYSIS

The analysis is an extension of the Myklestad-Prohl method, see Ref. 1, 2 and 3. The rotor, which is actually a continuous system with an infinite number of degrees of freedom, is replaced by a finite number of lumped masses connected by weightless springs. The computer program calculates the vibrational response of this equivalent system exactly.

Thus the accuracy of the results depends only on how closely the idealized system resembles the actual rotor.

Starting from the left end of the rotor, the program calculates step by step the bending moment, shear force, slope and deflection along the rotor. Neglecting the shear force contribution to the deflection, we get from Fig. 1:

(1) $$M_{n+1} = M'_n + L_n V_n$$

(2) $$\theta_{n+1} = \theta_n + a_n M'_n + b_n V_n$$

(3) $$X_{n+1} = X_n + L_n \theta_n + c_n M'_n + d_n V_n$$

where:

(4) $$a_n = \int_0^{L_n} \frac{d\xi}{EI} = \frac{L_n}{EI}$$ for EI constant in $0 \leq \xi \leq L_n$

(5) $$b_n = \int_0^{L_n} \frac{\xi d\xi}{EI} = \frac{L_n^2}{2EI}$$ " "

(6) $$c_n = L_n a_n - b_n = \frac{L_n^2}{2EI}$$ " "

(7) $$d_n = L_n b_n - \int_0^{L_n} \frac{\xi^2 d\xi}{EI} = \frac{L_n^3}{6EI}$$ " "

The program assumes EI constant between mass points. At the mass points, the forces acting on the rotor are introduced. Four contributions exist: (1) inertia force, (2) unbalance forces, (3) bearing reaction, and (4) gyroscopic moment. In general, not all 4 contributions apply to each mass point.

<u>Inertia force</u>. The rotor performs harmonic vibrations at the same frequency as the rotational speed. Thus the inertia force is:

(8) $$-m \frac{\partial^2 x}{\partial t^2} = m\omega^2 x$$

(9) $$-m \frac{\partial^2 y}{\partial t^2} = m\omega^2 y$$

Unbalance forces. To allow for change in circumferential position of the unbalance along the rotor, the unbalance is given two components U_x and U_y. This gives rise to an x and y component of the unbalance force:

(10) $$(V_{xn} - V_{x,n+1})_{unb.} = \omega^2 U_x \cos\omega t - \omega^2 U_y \sin\omega t$$

(11) $$(V_{y,n} - V_{y,n+1})_{unb.} = \omega^2 U_y \cos\omega t + \omega^2 U_x \sin\omega t$$

Bearing reaction. The bearing supports have flexibility and damping for both translatory and rotational motion of the rotor. Since the equations for the two types of motion are analogous, only the equations for translatory motion will be derived.

The bearing support is shown in Fig. 3. It consists of a pedestal with mass (M_{ox}, M_{oy}), supported by springs (γ_x, γ_y) and dashpots (σ_x, σ_y). There is no coupling between the x and y direction, i.e. no transfer impedance, nor between the translatory and rotational motion. The pedestal supports the bearing fluid film which is represented by 4 springs and 4 damping coefficients. If the relative motion between the journal center and the bearing housing is denoted (x', y'), then the bearing reaction becomes:

(12) $$\begin{aligned} (V_{xn} - V_{x,n-1})_{bearing} &= -K_{xx} x' - C_{xx} \dot{x}' - K_{xy} y' - C_{xy} \dot{y}' \\ (V_{yn} - V_{y,n-1})_{bearing} &= -K_{yx} x' - C_{yx} \dot{x}' - K_{yy} y' - C_{yy} \dot{y}' \end{aligned}$$

Setting:

$$x' = x'_c \cos\omega t + x'_s \sin\omega t$$
$$y' = y'_c \cos\omega t + y'_s \sin\omega t$$

we get from Newton's second law for the pedestal mass:

$$
(13)\quad
\begin{aligned}
&(K_{xx}+\gamma_x-\omega^2 M_{ox})X_c'+\omega(C_{xx}+\sigma_x)X_s'+K_{xy}y_c'+\omega C_{xy}y_s'=(\gamma_x-\omega^2 M_{ox})X_c+\omega\sigma_x X_s\\
&-\omega(C_{xx}+\sigma_x)X_c'+(K_{xx}+\gamma_x-\omega^2 M_{ox})X_s'-\omega C_{xy}y_c'+K_{xy}y_s'=-\omega\sigma_x X_c+(\gamma_x-\omega^2 M_{ox})X_s\\
&K_{yx}X_c'+\omega C_{yx}X_s'+(K_{yy}+\gamma_y-\omega^2 M_{oy})y_c'+\omega(C_{yy}+\sigma_y)\,y_s'=(\gamma_y-\omega^2 M_{oy})y_c+\omega\sigma_y y_s\\
&-\omega C_{yx}X_c'+K_{yx}X_s'-\omega(C_{yy}+\sigma_y)\,y_c'+(K_{yy}+\gamma_y-\omega^2 M_{oy})y_s'=-\omega\sigma_y y_c+(\gamma_y-\omega^2 M_{oy})y_s
\end{aligned}
$$

Solving the equations we obtain:

$$
(14)\quad
\begin{aligned}
(V_{xn}-V_{x,n-1})_{bearing}&=(-\Delta V_{ax}X_c-\Delta V_{bx}X_s-\Delta V_{cx}y_c-\Delta V_{dx}y_s)\cos\omega t\\
&\quad+(\Delta V_{bx}X_c-\Delta V_{ax}X_s+\Delta V_{dx}y_c-\Delta V_{cx}y_s)\sin\omega t\\
(V_{yn}-V_{y,n-1})_{bearing}&=(-\Delta V_{cy}X_c-\Delta V_{dy}X_s-\Delta V_{ay}y_c-\Delta V_{by}y_s)\cos\omega t\\
&\quad+(\Delta V_{dy}X_c-\Delta V_{cy}X_s+\Delta V_{by}y_c-\Delta V_{ay}y_s)\sin\omega t
\end{aligned}
$$

where:

$$
(15)\quad
\begin{aligned}
\Delta V_{ax}&=K_{xx}f+\omega C_{xx}g+K_{xy}q+\omega C_{xy}r\\
\Delta V_{bx}&=-K_{xx}g+\omega C_{xx}f-K_{xy}r+\omega C_{xy}q\\
\Delta V_{cx}&=K_{xx}h+\omega C_{xx}i+K_{xy}s+\omega C_{xy}t\\
\Delta V_{dx}&=-K_{xx}i+\omega C_{xx}h-K_{xy}t+\omega C_{xy}s\\
\Delta V_{ay}&=K_{yx}h+\omega C_{yx}i+K_{yy}s+\omega C_{yy}t\\
\Delta V_{by}&=-K_{yx}i+\omega C_{yx}h-K_{yy}t+\omega C_{yy}s\\
\Delta V_{cy}&=K_{yx}f+\omega C_{yx}g+K_{yy}q+\omega C_{yy}r\\
\Delta V_{dy}&=-K_{yx}g+\omega C_{yx}f-K_{yy}r+\omega C_{yy}q
\end{aligned}
$$

and:

$$f = \frac{GE + FD}{F^2 + G^2} \qquad g = \frac{GD - FE}{F^2 + G^2}$$

$$h = \frac{GJ + FH}{F^2 + G^2} \qquad i = \frac{GH - FJ}{F^2 + G^2}$$

$$F = A - K_{xy} Q + \omega C_{xy} R \qquad G = B - K_{xy} R - \omega C_{xy} Q$$

$$H = -K_{xy} S + \omega C_{xy} T \qquad J = -K_{xy} T - \omega C_{xy} S$$

$$Q = \frac{K_{yx} a + \omega C_{yx} b}{a^2 + b^2} \qquad R = \frac{\omega C_{yx} a - K_{yx} b}{a^2 + b^2}$$

$$S = \frac{ad + be}{a^2 + b^2} \qquad T = \frac{ae - bd}{a^2 + b^2}$$

$$A = K_{xx} + \gamma_x - \omega^2 M_{ox} \qquad B = \omega (C_{xx} + \sigma_x)$$

$$D = \gamma_x - \omega^2 M_{ox} \qquad E = \omega \sigma_x$$

$$a = K_{yy} + \gamma_y - \omega^2 M_{oy} \qquad b = \omega (C_{yy} + \sigma_y)$$

$$d = \gamma_y - \omega^2 M_{oy} \qquad e = \omega \sigma_y$$

$$q = -Qf - Rg \qquad r = -Qg + Rf$$

$$s = S - Qh - Ri \qquad t = -T - Qi + Rh$$

The equations for rotational motion are analogous to eq. (14) except for a sign reversal (sign convention, see Fig. 2):

$$(M'_{xn} - M_{xn})_{bearing} = (\Delta M_{ax}\theta_c + \Delta M_{bx}\theta_s + \Delta M_{cx}\varphi_c + \Delta M_{dx}\varphi_s)\cos\omega t + (-\Delta M_{bx}\theta_c + \Delta M_{ax}\theta_s - \Delta M_{dx}\varphi_c + \Delta M_{cx}\varphi_s)\sin\omega t$$

(16)

$$(M'_{yn} - M_{yn})_{bearing} = (\Delta M_{cy}\theta_c + \Delta M_{dy}\theta_s + \Delta M_{ay}\varphi_c + \Delta M_{by}\varphi_s)\cos\omega t + (-\Delta M_{dy}\theta_c + \Delta M_{cy}\theta_s - \Delta M_{by}\varphi_c + \Delta M_{ay}\varphi_s)\sin\omega t$$

where the coefficients $\Delta M_{ay}, \Delta M_{bx}$ etc. are computed from eq. (15) as $\Delta M_{ax} = \Delta V_{ax}$, $\Delta M_{bx} = \Delta V_{bx}$ etc. by replacing the translatory spring and damping coefficients by the corresponding rotational coefficients.

Since the fluid film coefficients are functions of speed, directly through the Sommerfeld number and indirectly through the decrease of eccentricity ratio with increasing speed, the computer program provides for expressing the coefficients as a function of speed, e.g.

$$K_{xx} = K_{xx,0} + K_{xx,1} \cdot \omega + K_{xx,2} \cdot \omega^2 \tag{18}$$

and similarly for the other coefficients. ω is the rotor speed in radians/sec.

Gyroscopic Moment. The gyroscopic moment derives from the change of the angular momentum vector of the rotating rotor mass as it whirls in an elliptical path around the steady state position of the rotor. For two special cases the gyroscopic moment is known:

(19)

circular whirl path: $M_{Gyr.} = (I_P - I_T)\,\omega^2\theta$

straight line (transverse vibrations): $M_{Gyr.} = -I_T\,\omega^2\theta$

where θ is the slope of the rotor deflection and I_P and I_T are the polar and transverse mass moment of inertia. For an elliptical path the gyroscopic moment is no longer linear with respect to the slope of the rotor, indicating that an elliptical path is actually not possible. However, in general the effect of the gyroscopic moment is not too big and for the present analysis an elliptical path will be assumed.

The coordinate system is shown in Fig. 4, where O is the steady state shaft center position and O' is the whirling shaft center. The moving coordinate system (ξ, η, ζ) is defined by its unit vectors:

$$\bar{e}_\xi = \left(\frac{\theta/\sqrt{\theta^2+\varphi^2}}{\sqrt{1+\theta^2+\varphi^2}} , \frac{\varphi/\sqrt{\theta^2+\varphi^2}}{\sqrt{1+\theta^2+\varphi^2}} , \frac{-\sqrt{\theta^2+\varphi^2}}{\sqrt{1+\theta^2+\varphi^2}} \right) \cong \left(\frac{\theta}{\sqrt{\theta^2+\varphi^2}} , \frac{\varphi}{\sqrt{\theta^2+\varphi^2}} ; \sqrt{\theta^2+\varphi^2} \right)$$

$$(20) \qquad \bar{e}_\eta = \left(\frac{-\varphi}{\sqrt{\theta^2+\varphi^2}} , \frac{\theta}{\sqrt{\theta^2+\varphi^2}} , 0 \right)$$

$$\bar{e}_\zeta = \left(\frac{\theta}{\sqrt{1+\theta^2+\varphi^2}} , \frac{\varphi}{\sqrt{1+\theta^2+\varphi^2}} , \frac{1}{\sqrt{1+\theta^2+\varphi^2}} \right) \cong \left(\theta , \varphi , 1 \right)$$

The angular velocity vector becomes:

$$(21) \qquad \bar{\omega} = \left(\omega_\xi , \omega_\eta , \omega_\zeta \right) = \left(\dot{e}_\eta e_\zeta , \dot{e}_\zeta e_\xi , \dot{e}_\xi e_\eta \right) = \left(\frac{\dot{\theta}\varphi - \theta\dot{\varphi}}{\sqrt{\theta^2+\varphi^2}} , \frac{\theta\dot{\theta}+\varphi\dot{\varphi}}{\sqrt{\theta^2+\varphi^2}} , -\frac{(\dot{\theta}\varphi-\theta\dot{\varphi})}{\theta^2+\varphi^2} \right)$$

The moment needed to sustain the motion is given by Eulers equations:

$$M_\xi = I_T \dot{\omega}_\xi + (I_P - I_T)\, \omega_\zeta \omega_\eta$$

$$(22) \qquad M_\eta = I_T \dot{\omega}_\eta + (I_P - I_T)\, \omega_\zeta \omega_\xi$$

$$M_\zeta = I_P \dot{\omega}_\zeta$$

where I denotes mass moment of inertia and $I_\xi = I_T$, $I_\eta = I_T$ and $I_\zeta = I_P$.

Let us first assume that (x, y) corresponds to the directions of the major and minor axis in the elliptical variation of the rotor slope. Then:

$$\theta_1 = E \cos(\omega t + \alpha)$$

$$(23)$$

$$\varphi_1 = G \sin(\omega t + \alpha)$$

Combining eq. (20), (21) and (22):

$$-M_x = -I_T \omega^2 \varphi_1 + I_P \omega^2 EG \left[2EG \frac{\varphi_1}{(\theta_1^2+\varphi_1^2)} + \frac{\frac{1}{\omega} \dot{\theta}_1}{\theta_1^2 + \varphi_1^2} \right]$$

$$(24)$$

$$M_y = -\ddot{I}_T \omega^2 \theta_1 + I_P \omega^2 EG \left[2EG \frac{\theta_1}{(\theta_1^2+\varphi_1^2)^2} - \frac{\frac{1}{\omega} \dot{\varphi}_1^2}{\theta_1^2 + \varphi_1^2} \right]$$

which clearly shows that the gyroscopic moment is not linear with respect to the rotor slope. However, only the first harmonic can do work on the rotor. Hence a Fourier analysis will be performed. The following integrals apply:

$$\int_0^{2\pi} \frac{\sin x \cos x \, dx}{E^2\cos^2 x + G^2 \sin^2 x} = 0$$

$$\int_0^{2\pi} \frac{\sin x \cos x \, dx}{(E^2\cos^2 x + G^2 \sin^2 x)^2} = 0$$

$$\int_0^{2\pi} \frac{\sin^2 x \, dx}{E^2\cos^2 x + G^2 \sin^2 x} = \frac{2\pi}{G(E+G)}$$

$$\int_0^{2\pi} \frac{\sin^2 x \, dx}{(E^2\cos^2 x + G^2 \sin^2 x)^2} = \frac{\pi}{EG^3}$$

$$\int_0^{2\pi} \frac{dx}{E^2\cos^2 x + G^2 \sin^2 x} = \frac{2\pi}{EG}$$

$$\int_0^{2\pi} \frac{dx}{(E^2\cos^2 x + G^2 \sin^2 x)^2} = \frac{\pi(E^2+G^2)}{E^3G^3}$$

Then the first harmonic becomes:

$$(24) \quad \begin{aligned} -M_x &= \left(\frac{2E}{E+G} I_P - I_T\right) \omega^2 \varphi_1 \\ M_y &= \left(\frac{2G}{E+G} I_P - I_T\right) \omega^2 \theta_1 \end{aligned}$$

In the limit, eqs. (24) agree with eqs. (19).

Eqs. (24) must be transformed back to the actual (θ,φ)-coordinate system. Setting

$$(25) \quad \begin{aligned} \theta &= \theta_c \cos \omega t + \theta_s \sin \omega t \\ \varphi &= \varphi_c \cos \omega t + \varphi_s \sin \omega t \end{aligned}$$

describing an elliptical variation of slope, we get:

(26)
$$\left.\begin{matrix}E\\G\end{matrix}\right\} = \sqrt{\frac{1}{2}\left(\theta_c^2+\theta_s^2+\varphi_c^2+\varphi_s^2\right) \pm \sqrt{\frac{1}{4}\left(\theta_c^2+\theta_s^2+\varphi_c^2+\varphi_s^2\right)^2-\left(\theta_c\varphi_s-\theta_s\varphi_c\right)^2}}$$

$$\cos 2\beta = \frac{\theta_c^2+\theta_s^2-\varphi_c^2-\varphi_s^2}{\sqrt{\left(\theta_c^2+\theta_s^2-\varphi_c^2-\varphi_s^2\right)^2+4\left(\theta_c\varphi_c+\theta_s\varphi_s\right)^2}}$$

$$\sin 2\beta = \frac{2\left(\theta_c\varphi_c+\theta_s\varphi_s\right)}{\sqrt{\left(\theta_c^2+\theta_s^2-\varphi_c^2-\varphi_s^2\right)^2+4\left(\theta_c\varphi_c+\theta_s\varphi_s\right)^2}}$$

where β is the angle from the position χ-axis to the major axis E, position in the same direction as ω . Then:

(27)
$$\theta_1 = \theta\cos\beta + \varphi\sin\beta$$
$$\varphi_1 = -\theta\sin\beta + \varphi\cos\beta$$

Substituting eq. (26)-(27) into eqs. (24) gives:

(28)
$$M_y = \left(M_{xn}' - M_{xn}\right)_{Gyro} = \omega^2\left[2\Delta M_{Gx} I_P - \theta_c I_T\right]\cos\omega t + \omega^2\left[2\Delta M_{Gy} I_P - \theta_s I_T\right]\sin\omega t$$

(29)
$$-M_x = \left(M_{yn}' - M_{yn}\right)_{Gyro} = \omega^2\left[-2\Delta M_{Gy} I_P - \varphi_c I_T\right]\cos\omega t + \omega^2\left[2\Delta M_{Gx} I_P - \varphi_s I_T\right]\sin\omega t$$

where

(30)
$$\Delta M_{Gx} = \frac{(\theta_c+\varphi_s)(\theta_c\varphi_s-\theta_s\varphi_c)}{(\theta_c+\varphi_s)^2+(\theta_s-\varphi_c)^2}$$

(31)
$$\Delta M_{Gy} = \frac{(\theta_s-\varphi_c)(\theta_c\varphi_s-\theta_s\varphi_c)}{(\theta_c+\varphi_s)^2+(\theta_s-\varphi_c)^2}$$

Since eq. (28) and eq. (29) are not linear, an iterative method is used. For each rotor speed, the program performs a number of iterations. The first iteration is done without gyroscopic moment. After the first iteration, the gyroscopic moment is calculated from eq. (28)-(29) and these values are used in the second iteration and so on. The calculation has converged when the relative change in rotor amplitude and slope between two iterations is smaller than a specified limit.

EQUATIONS FOR ROTOR CALCULATION

The bending moment, the shear force, the slope and the deflection are expressed by:

$$M_x = M_{xc} \cos \omega t + M_{xs} \sin \omega t$$

$$V_x = V_{xc} \cos \omega t + V_{xs} \sin \omega t$$

$$\theta = \theta_c \cos \omega t + \theta_s \sin \omega t$$

$$x = x_c \cos \omega t + x_s \sin \omega t$$

and similarly for the y-direction. Then eq. (1), (2), (3), (8), (9), (10), (11), (14), (16), (17), and (29) may be combined to give the equations used in the rotor calculation (see Fig. 2):

$$M'_{xcn} = M_{xcn} + \Delta M_{axn}\theta_{cn} + \Delta M_{bxn}\theta_{sn} + \Delta M_{cxn}\varphi_{cn} + \Delta M_{dxn}\varphi_{sn} + (M'_{xcn} - M_{xcn})_{Gyro}$$

$$M'_{xsn} = M_{xsn} - \Delta M_{bxn}\theta_{cn} + \Delta M_{axn}\theta_{sn} - \Delta M_{dxn}\varphi_{cn} + \Delta M_{cxn}\varphi_{sn} + (M'_{xsn} - M_{xsn})_{Gyro}$$

$$M'_{ycn} = M_{ycn} + \Delta M_{cyn}\theta_{cn} + \Delta M_{dyn}\theta_{sn} + \Delta M_{ayn}\varphi_{cn} + \Delta M_{byn}\varphi_{sn} + (M'_{ycn} - M_{ysn})_{Gyro}$$

$$M'_{ysn} = M_{ysn} - \Delta M_{dyn}\theta_{cn} + \Delta M_{cyn}\theta_{sn} - \Delta M_{byn}\varphi_{cn} + \Delta M_{ayn}\varphi_{sn} + (M'_{ysn} - M_{ysn})_{Gyro}$$

$$V_{xcn} = V_{xc,n-1} + \left[m_n\omega^2 - \Delta V_{axn}\right] x_{cn} - \Delta V_{bxn} x_{sn} - \Delta V_{cxn} y_{cn} - \Delta V_{dxn} y_{sn} + \omega^2 U_{xn}$$

$$V_{xsn} = V_{xs,n-1} + \Delta V_{bxn} x_{cn} + \left[m_n\omega^2 - \Delta V_{axn}\right] x_{sn} + \Delta V_{dxn} y_{cn} - \Delta V_{cxn} y_{sn} - \omega^2 U_{yn} \tag{32}$$

$$V_{ycn} = V_{yc,n-1} - \Delta V_{cyn} x_{cn} - \Delta V_{dyn} x_{sn} + \left[m_n\omega^2 - \Delta V_{ayn}\right] y_{cn} - \Delta V_{byn} y_{sn} + \omega^2 U_{yn}$$

$$V_{ysn} = V_{ys,n-1} + \Delta V_{dyn} x_{cn} - \Delta V_{cyn} x_{sn} + \Delta V_{byn} y_{cn} + \left[m_n\omega^2 - \Delta V_{ayn}\right] y_{sn} + \omega^2 U_{xn}$$

$$M_{xc,n+1} = M'_{xcn} + L_n V_{xcn}$$

$$M_{xs,n+1} = M'_{xsn} + L_n V_{xsn}$$

$$M_{yc,n+1} = M'_{ycn} + L_n V_{ycn}$$

$$M_{ys,n+1} = M'_{ysn} + L_n V_{ysn}$$

$$\theta_{c,n+1} = \theta_{cn} + a_n M'_{xcn} + b_n V_{xcn}$$

$$\theta_{s,n+1} = \theta_{sn} + a_n M'_{xsn} + b_n V_{xsn}$$

$$\varphi_{c,n+1} = \varphi_{cn} + a_n M'_{ycn} + b_n V_{ycn}$$

$$\varphi_{s,n+1} = \varphi_{sn} + a_n M'_{ysn} + b_n V_{ysn}$$

$$X_{c,n+1} = X_{cn} + L_n \theta_{cn} + b_n M'_{xcn} + d_n V_{xcn}$$

$$X_{s,n+1} = X_{sn} + L_n \theta_{sn} + b_n M'_{xsn} + d_n V_{xsn}$$

$$y_{c,n+1} = y_{cn} + L_n \varphi_{cn} + b_n M'_{ycn} + d_n V_{ycn}$$

$$y_{s,n+1} = y_{sn} + L_n \varphi_{sn} + b_n M'_{ysn} + d_n V_{ysn}$$

In the above equations a_n, b_n, d_n are given by eq. (4), (5) and (7), $\Delta M_{axn}, \Delta M_{bxn}$ -------- ΔM_{dyn} and ΔV_{axn}, ΔV_{bxn} ------ ΔV_{dyn} by eqs. (15) and $(M'_{xcn} - M_{xcn})_{Gyro}$ --------- $(M'_{ysn} - M_{ysn})_{Gyro}$ by eq. (28)-(29).

Boundary Conditions. The rotor is assumed to have free ends. No loss in generality occurs by this condition since it may be changed by letting the end points have bearing support. A proper choice of support coefficients will then allow for any type of end conditions.

For a rotor with free ends the bending moment and the shear force are zero at the end:

(33) $M_{xc1} = M_{xs1} = M_{yc1} = M_{ys1} = V_{xc1} = V_{xs1} = V_{yc1} = V_{ys1} = 0$

(34) $M'_{xcr} = M'_{xsr} = M'_{ycr} = M'_{ysr} = V_{xcr} = V_{xsr} = V_{ycr} = V_{ysr} = 0$

Starting from the left end of the rotor (see Fig. 2), eq. (33) is used. However, the slope and the deflection are unknown. Using the superposition principle, each unknown is applied separately. A summation gives the combined effect. Ten calculations are performed, using eqs. (32).

1.	$\theta_{c1} = 1$	$\theta_{s1} = \varphi_{c1} = \varphi_{s1} = x_{c1} = x_{s1} = y_{c1} = y_{s1} = U_{xn} = U_{yn} = 0$
2.	$\theta_{s1} = 1$	$\theta_{c1} = \varphi_{c1} = \varphi_{s1} = x_{c1} = x_{s1} = y_{c1} = y_{s1} = U_{xn} = U_{yn} = 0$
3.	$\varphi_{c1} = 1$	$\theta_{c1} = \theta_{s1} = \varphi_{s1} = x_{c1} = x_{s1} = y_{c1} = y_{s1} = U_{xn} = U_{yn} = 0$
4.	$\varphi_{s1} = 1$	$\theta_{c1} = \theta_{s1} = \varphi_{c1} = x_{c1} = x_{s1} = y_{c1} = y_{s1} = U_{xn} = U_{yn} = 0$
5.	$x_{c1} = 1$	$\theta_{c1} = \theta_{s1} = \varphi_{c1} = \varphi_{s1} = x_{s1} = y_{c1} = y_{s1} = U_{xn} = U_{yn} = 0$
6.	$x_{s1} = 1$	$\theta_{c1} = \theta_{s1} = \varphi_{c1} = \varphi_{s1} = x_{c1} = y_{c1} = y_{s1} = U_{xn} = U_{yn} = 0$
7.	$y_{c1} = 1$	$\theta_{c1} = \theta_{s1} = \varphi_{c1} = \varphi_{s1} = x_{c1} = x_{s1} = y_{s1} = U_{xn} = U_{yn} = 0$
8.	$y_{s1} = 1$	$\theta_{c1} = \theta_{s1} = \varphi_{c1} = \varphi_{s1} = x_{c1} = x_{s1} = y_{c1} = U_{xn} = U_{yn} = 0$
9.	$U_{xn} = U_{xn} \quad U_{yn} = U_{yn}$	$\theta_{c1} = \theta_{s1} = \varphi_{c1} = \varphi_{s1} = x_{c1} = x_{s1} = y_{c1} = y_{s1} = 0$
10.	Gyroscopic moment applied	$\theta_{c1} = \theta_{s1} = \varphi_{c1} = \varphi_{s1} = x_{c1} = x_{s1} = y_{c1} = y_{s1} = U_{xn} = U_{yn} = 0$

For each calculation eqs. (32) are used to calculate the bending moment, the shear force, the slope and the deflection along the rotor. At the right rotor end, station r, eq. (34) must be satisfied, i.e.

$$
\left\{\begin{matrix}
M'_{xcr,1} & M'_{xcr,2} & \cdots & M'_{xcr,8} \\
M'_{xsr,1} & M'_{xsr,2} & \cdots & M'_{xsr,8} \\
M'_{ycr,1} & M'_{ycr,2} & \cdots & M'_{ycr,8} \\
M'_{ysr,1} & M'_{ysr,2} & \cdots & M'_{ysr,8} \\
V_{xcr,1} & V_{xcr,2} & \cdots & V_{xcr,8} \\
V_{xsr,1} & V_{xsr,2} & \cdots & V_{xsr,8} \\
V_{ycr,1} & V_{ycr,2} & \cdots & V_{ycr,8} \\
V_{ysr,1} & V_{ysr,2} & \cdots & V_{ysr,8}
\end{matrix}\right\}
\left\{\begin{matrix}
\theta_{c1} \\ \theta_{s1} \\ \varphi_{c1} \\ \varphi_{s1} \\ x_{c1} \\ x_{s1} \\ y_{c1} \\ y_{s1}
\end{matrix}\right\}
=
\left\{\begin{matrix}
-M'_{xcr,9} - M'_{xcr,10} \\
-M'_{xsr,9} - M'_{xsr,10} \\
-M'_{ycr,9} - M'_{ycr,10} \\
-M'_{ysr,9} - M'_{ysr,10} \\
-V_{xcr,9} - V_{xcr,10} \\
-V_{xsr,9} - V_{xsr,10} \\
-V_{ycr,9} - V_{ycr,10} \\
-V_{ysr,9} - V_{ysr,10}
\end{matrix}\right\}
\tag{35}
$$

Eqs. (35) are then solved for $\theta_{c1}, \theta_{s1} \cdots y_{s1}$, and the actual values of bending moment, shear force etc, along the rotor can be determined. At a given rotor speed, eqs. (35) are first solved without gyroscopic moment, i.e. $M'_{xcr,10} = ------ = V_{ysr,10} = 0$. Then the gyroscopic moment is applied according to eq. (28)-(29) and new values are found for $\theta_{c1}, \theta_{s1}, ------ y_{s1}$ from eqs. (35). This process is repeated until at the k'th iteration:

$$(36) \quad \frac{|\theta_{c1}^{(k)} - \theta_{c1}^{(k-1)}| + |\theta_{s1}^{(k)} - \theta_{s1}^{(k-1)}| + |\varphi_{c1}^{(k)} - \varphi_{c1}^{(k-1)}| + ------ + |y_{s1}^{(k)} - y_{s1}^{(k-1)}|}{|\theta_{c1}^{(k)}| + |\theta_{s1}^{(k)}| + |\varphi_{c1}^{(k)}| + ------- |y_{s1}^{(k)}|} \leq \varepsilon_{Gyro}$$

where ε_{Gyro} is the convergence limit specified by the computer input. If the calculation does not converge within a specified number of iterations, the program goes on to a new rotor speed.

In the computer output, the rotor deflection is given by the dimensions of the elliptical whirl path. We have:

$$(37) \quad \begin{aligned} x &= x_c \cos\omega t + x_s \sin\omega t \\ y &= y_c \cos\omega t + y_s \sin\omega t \end{aligned}$$

As shown in Fig. 5, the (x,y)-coordinate system is rotated an angle β in the same direction as ω to become (x_1, y_1). Then

$$(38) \quad \begin{aligned} x_1 &= a\cos(\omega t + \alpha) \\ y_1 &= b\sin(\omega t + \alpha) \end{aligned}$$

where a and b are the major and minor axis respectively of the ellipse. From Fig. 5:

$$\begin{aligned} x_1 &= x\cos\beta + y\sin\beta \\ y_1 &= -x\sin\beta + y\cos\beta \end{aligned}$$

Then:

$$(39) \quad \left.\begin{matrix} a \\ b \end{matrix}\right\} = \sqrt{\frac{1}{2}\left[(x_c^2 + x_s^2 + y_c^2 + y_s^2) \pm \sqrt{(x_c^2 + x_s^2 - y_c^2 - y_s^2)^2 + 4(x_c y_c + x_s y_s)^2}\right]}$$

Here it is necessary to allow b to become negative. The reason is that the transformation from the x-y-coordinates to the ellipse must be able to discern between forward and backward whirl (i.e. the shaft center may travel in the same direction or in the opposite direction of the direction of rotation depending on the values of x_c, x_s, y_c and y_s). Let the angle between the x-axis and the instantaneous radius vector be γ :

$$\gamma = \tan^{-1}\left(\frac{y}{x}\right)$$

Then:

$$\dot{\gamma} = \frac{x\dot{y} - \dot{x}y}{x^2+y^2} = \frac{\omega[x_c y_s - x_s y_c]}{x^2+y^2}$$

i.e.

$$(x_c y_s - x_s y_c) > 0 \quad : \text{forward whirl}$$

$$(x_c y_s - x_s y_c) < 0 \quad : \text{backward whirl}$$

$$(x_c y_s - x_s y_c) = 0 \quad : \text{straight line orbit } (b=0)$$

Therefore:

(39a) $$b = \frac{(x_c y_s - x_s y_c)}{|x_c y_s - x_s y_c|}\sqrt{\tfrac{1}{2}\left[(x_c^2+x_s^2+y_c^2+y_s^2) - \sqrt{(x_c^2+x_s^2-y_c^2-y_s^2)^2+4(x_c y_c+x_s y_s)^2}\,\right]}$$

To find α and β expand Eq.(38)

(a) $a\cos\alpha = x_c\cos\beta + y_c\sin\beta$

(b) $-a\sin\alpha = x_s\cos\beta + y_s\sin\beta$

(c) $b\sin\alpha = -x_c\sin\beta + y_c\cos\beta$

(d) $b\cos\alpha = -x_s\sin\beta + y_s\cos\beta$

Then:

$(a)^2+(b)^2+(c)^2+(d)^2 :\quad a^2+b^2 = x_c^2+x_s^2+y_c^2+y_s^2$

$(a)^2+(b)^2-(c)^2-(d)^2 :\quad a^2-b^2 = (x_c^2+x_s^2-y_c^2-y_s^2)\cos 2\beta + 2(x_c y_c + x_s y_s)\sin 2\beta$

i.e. $\cos 2\beta = \dfrac{x_c^2+x_s^2-y_c^2-y_s^2}{a^2-b^2}$ $\qquad \sin 2\beta = \dfrac{2(x_c y_c + x_s y_s)}{a^2-b^2}$

Next:

$-(a)\cdot(b) - (c)\cdot(d) :\quad \tfrac{1}{2}(a^2-b^2)\sin 2\alpha = -(x_c x_s + y_c y_s)$

$(a)^2-(b)^2+(c)^2-(d)^2 :\quad (a^2-b^2)\cos 2\alpha = x_c^2 - x_s^2 + y_c^2 - y_s^2$

i.e. $\cos 2\alpha = \dfrac{x_c^2-x_s^2+y_c^2-y_s^2}{a^2-b^2}$ $\qquad \sin 2\alpha = \dfrac{-2(x_c x_s + y_c y_s)}{a^2-b^2}$

Thus in total:

(40) $$\tan 2\beta = \frac{2(x_c y_c + x_s y_s)}{x_c^2 + x_s^2 - y_c^2 - y_s^2} \qquad \cos 2\beta = \frac{x_c^2 + x_s^2 - y_c^2 - y_s^2}{a^2 - b^2}$$

(41) $$\tan 2\alpha = \frac{-2(x_c x_s + y_c y_s)}{x_c^2 - x_s^2 + y_c^2 - y_s^2} \qquad \cos 2\alpha = \frac{x_c^2 - x_s^2 + y_c^2 - y_s^2}{a^2 - b^2}$$

Thus β is the angle from the positive x-axis to the major axis of the ellipse (positive with ω) and α is the phase angle for the radius vector, measured positive from the major axis in the direction of ω. The computer output gives a, b, β and α for both the deflection and the bending moment.

Coupling Stations. The programs allow for couplings in the rotor. At these stations, the bending moment vanishes, i.e. $M_n^1 = 0$ (the coupling point is taken just to the right of the mass station). When the program encounters a coupling station, say station i, the following equations are set up:

(42) $$\begin{bmatrix} M_{xci,1} & M_{xci,2} & \cdots & M_{xci,4} \\ M_{xsi,1} & M_{xsi,2} & \cdots & M_{xsi,4} \\ M_{yci,1} & M_{yci,2} & \cdots & M_{yci,4} \\ M_{ysi,1} & M_{ysi,2} & \cdots & M_{ysi,4} \end{bmatrix} \begin{bmatrix} \theta_{c1} \\ \theta_{s1} \\ \varphi_{c1} \\ \varphi_{s1} \end{bmatrix} = - \begin{bmatrix} M'_{xci,5} & \cdots & M'_{xci,8} \\ M'_{xsi,5} & \cdots & M'_{xsi,8} \\ M'_{yci,5} & \cdots & M'_{yci,8} \\ M'_{ysi,5} & \cdots & M'_{ysi,8} \end{bmatrix} \begin{bmatrix} x_{c1} \\ x_{s1} \\ y_{c1} \\ y_{s1} \end{bmatrix} - \begin{bmatrix} M'_{xci,9} + M'_{xci,10} \\ M'_{xsi,9} + M'_{xsi,10} \\ M'_{yci,9} + M'_{yci,10} \\ M'_{ysi,9} + M'_{ysi,10} \end{bmatrix}$$

or upon solving

(43) $$\theta_i = a_{ij} x_j + b_i \qquad (i, j = 1, 2, 3, 4)$$

where $\theta_1 = \theta_{c1}, \theta_2 = \theta_{c2}$ etc. and $x_1 = x_{c1}, x_2 = x_{s1}$ etc. Then the bending moment, shear force, slope and deflection before the coupling station become functions of x_{c1}, x_{s1}, y_{c1} and y_{s1} only. As an example, let the shear force at a station be:

$$V_{xcn} = V_1 \theta_{c1} + V_2 \theta_{s1} + V_3 \varphi_{c1} + V_4 \varphi_{s1} + V_5 x_{c1} + V_6 x_{s1} + V_7 y_{c1} + V_8 y_{s1} + V_9 + V_{10}$$

Introducing eq. (43) gives:

$$
\begin{aligned}
(44)\qquad V_{xcn} = {} & [V_5 + a_{11}V_1 + a_{21}V_2 + a_{31}V_3 + a_{41}V_4]\,x_{c1} + [V_6 + a_{12}V_1 + a_{22}V_2 + a_{32}V_3 + a_{42}V_4]\,x_{s1} \\
& + [V_7 + a_{13}V_1 + a_{23}V_2 + a_{33}V_3 + a_{43}V_4]\,y_{c1} + [V_8 + a_{14}V_1 + a_{24}V_2 + a_{34}V_3 + a_{44}V_4]\,y_{s1} \\
& + [V_9 + V_{10} + b_1V_1 + b_2V_2 + b_3V_3 + b_4V_4]
\end{aligned}
$$

and similarly for $V_{xsn}, V_{ycn}, V_{ysn}, M_{xcn}, ------ y_{sn}$. Instead of $\Theta_{c1}, \Theta_{s1}, \varphi_{c1}$ and φ_{s1} we have as new variables the slopes just to the right of the coupling station m, i.e. $\Theta_{cm}, \Theta_{sm}, \varphi_{cm}, \varphi_{sm}$. Then the calculation proceeds as before until either a new coupling station or the right end of the rotor is reached.

Transmitted Force and Pedestal Motion

Let the force transmitted to the pedestal at station n be denoted F. From Eq. (12) it is seen:

$$
(45)\qquad
\begin{aligned}
F_{xc} &= V_{xc,n-1} - V_{xcn} + m_n\omega^2 x_{cn} + \omega^2 U_{xn} \\
F_{xs} &= V_{xs,n-1} - V_{xsn} + m_n\omega^2 x_{sn} - \omega^2 U_{yn} \\
F_{yc} &= V_{yc,n-1} - V_{ycn} + m_n\omega^2 y_{cn} + \omega^2 U_{yn} \\
F_{ys} &= V_{ys,n-1} - V_{ysn} + m_n\omega^2 y_{sn} + \omega^2 U_{xn}
\end{aligned}
$$

Denoting the amplitude of the pedestal masses x_p and y_p (see Fig. 3) we get:

$$
\begin{aligned}
x_p &= \frac{F_{xc} - iF_{xs}}{\varkappa_x - M_x\omega^2 + i\omega\delta_x} \\
y_p &= \frac{F_{yc} - iF_{ys}}{\varkappa_y - M_y\omega^2 + i\omega\delta_y}
\end{aligned}
$$

or

$$
(46)\qquad
\begin{aligned}
x_{pc} &= \frac{F_{xc}(\varkappa_x - M_x\omega^2) - F_{xs}\,\omega\delta_x}{(\varkappa_x - M_x\omega^2)^2 + (\omega\delta_x)^2} \\
x_{ps} &= \frac{F_{xc}\,\omega\delta_x + F_{xs}(\varkappa_x - M_x\omega^2)}{(\varkappa_x - M_x\omega^2)^2 + (\omega\delta_x)^2} \\
y_{pc} &= \frac{F_{yc}(\varkappa_y - M_y\omega^2) - F_{ys}\,\omega\delta_y}{(\varkappa_y - M_y\omega^2)^2 + (\omega\delta_y)^2} \\
y_{ps} &= \frac{F_{yc}\,\omega\delta_y + F_{ys}(\varkappa_y - M_y\omega^2)}{(\varkappa_y - M_y\omega^2)^2 + (\omega\delta_y)^2}
\end{aligned}
$$

The force transmitted to the base becomes:

$$
\begin{aligned}
P_x &= \varkappa_x x_p + i\omega\delta_x x_p \\
P_y &= \varkappa_y y_p + i\omega\delta_y y_p
\end{aligned}
$$

or:

$$
(47)\quad
\begin{aligned}
P_{xc} &= F_{xc} + M_x \omega^2 x_{pc} \\
P_{xs} &= F_{xs} + M_x \omega^2 x_{ps} \\
P_{yc} &= F_{yc} + M_y \omega^2 y_{pc} \\
P_{ys} &= F_{ys} + M_y \omega^2 y_{ps}
\end{aligned}
$$

Energy Balance

Let the relative amplitude between rotor and pedestal mass be $x' = x - x_p$ and $y' = y - y_p$ at a bearing station. Then the energy dissipated in the bearing and the pedestal per revolution becomes:

(48) Energy Dissipated =

$$
\begin{aligned}
&\pi\{\omega C_{xx}(x_c'^2 + x_s'^2) + \omega C_{yy}(y_c'^2 + y_s'^2) + (\omega C_{xy} + \omega C_{yx})(x_c' y_c' + x_s' y_s') \\
&\quad - (K_{xy} - K_{yx})(x_c' y_s' - x_s' y_c') + \omega \delta_x (x_{pc}^2 + x_{ps}^2) + \omega \delta_y (y_{pc}^2 + y_{ps}^2)\} \\
&+ \pi\{\omega D_{xx}(\theta_c'^2 + \theta_s'^2) + \omega D_{yy}(\varphi_c'^2 + \varphi_s'^2) + (\omega D_{xy} + \omega D_{yx})(\theta_c' \varphi_c' + \theta_s' \varphi_s') \\
&\quad - (M_{xy} - M_{yx})(\theta_c' \varphi_s' - \theta_s' \varphi_c') + \omega \delta_x (\theta_{pc}^2 + \theta_{ps}^2) + \omega \delta_y (\varphi_{pc}^2 + \varphi_{ps}^2)\}
\end{aligned}
$$

A summation over all bearings gives the total dissipated energy. At each unbalance station there is an energy input:

(49) Energy Input: $\pi\left\{ \omega^2 U_x (x_s - y_c) + \omega^2 U_y (x_c + y_s) \right\}$

Summing over all unbalance stations gives the total energy input which must equal the dissipated energy.

COMPUTER INPUT

The input data is prepared according to the following instructions. Note that, unless specifically stated, no input card may be omitted.

Card 1 and 2: (72 cols. Hollerith) Identification:- Any descriptive text may be punched in cols. 2-72. These two cards must always be included.

Card 3: (10I5) Control parameters -

Word 1. Number of rotor mass stations - The number of mass stations is determined by the above considerations. Also, there must be a mass station at each rotor end, at each bearing, at each unbalance and at each coupling point. The mass at a station may be zero. The maximum number of mass stations is 80.

Word 2. Number of bearings - This integer denotes the total number of bearings along the rotor. A maximum of 25 bearings is possible.

Word 3. Number of unbalance stations - This integer gives the total number of mass stations at which unbalance is applied. A maximum of 80 unbalance stations is possible.

Word 4. Number of coupling stations - This integer gives the total number of coupling points. It cannot exceed 20.

Word 5. Pedestal flexible/rigid - If this integer is zero, the program assumes the pedestal to be rigid for both translatory and rotational motion and no pedestal data is included. If the integer is 1, the pedestal has flexibility and damping and pedestal data must be provided.

Word 6. Support tilting - If this integer is zero, neither the bearings nor the pedestals resist rotation. In that case, neither the input for the bearing dynamic coefficients for rotational motion nor the pedestal data for rotational motion can be included. If the integer is 1, the bearings and the pedestals have flexibility and damping for rotational motion.

Word 7. Gyroscopic moment - If this integer is zero, no gyroscopic moment is included in the calculation. If gyroscopic moment is desired, the integer should be 1.

Word 8. Number of computations - It was indicated above that the eight bearing parameters were dynamic coefficients and so could account for the variation of parameters with running speed in an approximate manner. However, if a more precise representation of these parameters is desired, these values can be entered each time a new running speed is designated. In order to facilitate this, there is provision in the program for entering only the bearing or bearing and pedestal data and the corresponding running speed without re-entering the rotor, coupling or unbalance data. Then this word 8 of the control parameters designates the number of sets of parameters which are to be run. If this value is 1, the program assumes that the bearing data is being entered as coefficients of quadratic equations in ω . Note below that the input format of the bearing data differs depending on whether this value is equal to or greater than one.

Word 9. Diagnostic - If this integer is zero, no diagnostic will be performed. A value of 1 will provide the diagnostic output: the diagnostic increases the amount of output a considerable amount and is provided primarily for use in trouble-shooting the program and so this value should always be zero.

Word 10. Input - If this integer is zero, the program will return to read in a new set of input upon completion of the computation. For the last set of input this value should be 1.

Card 4. (1P4E15.7)

Word 1 is Young' modulus E in lbs/in^2. It is constant throughout the rotor. Since the program never uses E by itself but always in the product EI (I=cross-sectional moment of inertia) any actual variation in E can be absorbed by changing I accordingly.

Word 2 is the scale factor for the determinant in the simultaneous equation subroutine. In general this item is 1.0. It is a factor by which the determinant is multiplied to control computer over/underflow. The simultaneous equation subroutine is used 4 places in the program: once when solving for the unknown end deflections (i.e.Eq.(35)) and 3 times when solving for the unkown slopes in the coupling calculation (i.e. Eq. (42)). If an over/underflow occurs during the calculation the program output will contain: "OVER/UNDERFLOW IN XSIMEQF AT _ _ (integer)" where the value of the integer is 1 to 4. If it is 1, 2 or 3 the error is in the coupling calculation. If it is 4 the error is in solving Eq. (35). Changing the scale-factor may eliminate the trouble.

If the determinant is singular the output gives: "MATRIX IS SINGULAR IN XSIMEQF AT _ _(integer)". If either of the two errors occur the program proceeds with a new rotor speed.

ROTOR DATA

The rotor data will differ depending on whether the effect of the gyroscopic moment is included in the computation. For the case where no gyroscopic moment is included; i.e. where word 7 of card 3 is zero, the rotor data is entered as follows:

Card: (1P3E14.6) - An input card must be given for each mass station. Each card has 3 items.

Word 1 - the mass at the station in lbs.

Word 2 - the length of the shaft section to the right of the station in inches.

Word 3 - the cross-sectional moment of inertia of the shaft section to the right of the station in in^4.

For the last mass station the shaft length and the cross-sectional moment of inertia has no meaning and may be set equal to zero.

If gyroscopic motion is included and word 7, card 3, is not equal to zero, then each card contains two more items in addition to the 3 items indicated just above. Also for this case, the rotor data cards are immediately preceded by a card which contains two values defined as follows:

Card: (I5,1PE23.6). Gyroscopic moment parameters -

Word 1 - Number of iterations - For each rotor speed the program first calculates the unbalance response without gyroscopic moment. Based on the thus obtained rotor slopes, the gyroscopic moment is computed and applied to the rotor, resulting in new values of the slope and the process is repeated. The program counts the number of iterations, excluding the calculation without gyroscopic moment. If the count exceeds this input item, the results obtained are printed out, the iteration count is reset to 1 and a new rotor speed calculation starts.

Word 2 - Convergence limit - After each gyroscopic moment iteration, the following relative error is calculated:

$$\frac{|\theta_{c1}^{(K)} - \theta_{c1}^{(K-1)}| + |\theta_{s1}^{(K)} - \theta_{s1}^{(K-1)}| + \cdots + |y_{s1}^{(K)} - y_{s1}^{(K-1)}|}{|\theta_{c1}^{(K)}| + |\theta_{s1}^{(K)}| + |\varphi_{c1}^{(K)}| + \cdots + |y_{s1}^{(K)}|}$$

where $\theta_{c1}, \theta_{s1}, \varphi_{c1}$ and φ_{s1} are the slopes and x_{c1}, x_{s1}, y_{c1} and y_{s1} are the deflections at the left rotor end. The superscript is the iteration number. For each iteration the computer output gives the iteration number and the error. When the error is less than or equal to the input convergence limit, the program prints the results, resets the iteration count to 1 and proceeds with a new rotor speed.

Following this card are the rotor data cards.

Card: (1P5E14.6). An input card is required for each mass station. Each card contains 5 items; the first 3 words are the same as those for the non-gyroscopic moment case above and the remaining two are:

Word 4 - the polar mass moment of inertia in lbs.in^2

Word 5 - the transverse mass moment of inertia in lbs.in^2

LOCATION OF BEARING SUPPORTS

Card: (14I5). This list provides the numbers of the mass stations at which there is a bearing.

UNBALANCE DATA

Card: (I5, 1P2E15.7). A card is provided for each of the unbalance stations. Each card contains 3 values:

Word 1 - an integer which denotes the number of the mass station at which the unbalance applies.

Word 2 - the cosine component of the unbalance in oz. in.

Word 3 - the sine component of the unbalance in oz. in.

By providing two unbalance components, it is possible to take into consideration the circumferential variation of unbalance along the rotor.

COUPLING DATA

If the rotor does not contain couplings, (Word 4, card 3 is zero), then no coupling data is necessary. If word 4, card 3 is not zero, the following card must be included.

Card: (14I5) - This is a list of integers denoting the mass stations at which there is a coupling.

PEDESTAL DATA

If the pedestal is considered to be infinitely rigid, then no pedestal data is required. In this case word 5, card 3, pedestal flexible/rigid is zero. Otherwise, pedestal data is required. The pedestal data, like the bearing data, is separated into translatory and rotational parameters. Also, as before, the control parameter is the word 6, card 3, support tilting.

Card: (1P6E12.4) - A card must be provided for each bearing. On it are 6 values as follows:

Word 1 - the weight of the pedestal in the x coordinate in lbs.

Word 2 - the pedestal stiffness along the x coordinate in lbs/in.

Word 3 - the pedestal damping along the x coordinate in lbs-sec/in.

Word 4 - same as word 1 but for the y coordinate.

Word 5 - same as word 2 but for the y coordinate.

Word 6 - same as word 3 but for the y coordinate.

If word 6, card 3, support tilting, is not zero, then all of the cards concerned with the translatory parameters are followed by the cards for the rotational parameters. Again there are 6 values on each card as follows:

Word 1 - the mass moment of inertia of the pedestal mass, associated with the x coordinate in lbs.in^2

Word 2 - the pedestal spring coefficient for rotational motion, associated with the x coordinate in lbs-in/rad.

Word 3 - the pedestal damping coefficient for rotational motion, associated with the x coordinate in lbs-in.-sec/rad.

Word 4 - same as word 1 but associated with the y coordinate.

Word 5 - same as word 2 but associated with the y coordinate.

Word 6 - same as word 3 but associated with the y coordinate.

SPEED AND BEARING DATA

Each bearing is represented by 16 dynamic coefficients, 8 for translatory motion and 8 for rotational motion. Of the 8 coefficients, 4 are spring coefficients and 4 are damping coefficients. Since the coefficients in general change with speed, each coefficient is expressed by three components; e.g.

$$K_{xx} = K_{xx,0} + K_{xx,1}\,\omega + K_{xx,2}\,\omega^2$$

where ω is the rotor speed in rad/sec, $K_{xx,0}$ in lbs/in., $K_{xx,1}$ in lbs-sec/in.rad. and $K_{xx,2}$ in lbs-sec^2/in.rad^2. Similar equations hold for the other 15 coefficients. As indicated earlier, if it is desired to enter the bearing data at each value of frequency, there is provision for this in the program.

If word 8, card 3 is 1, the program assumes the bearing data is provided as frequency dependent coefficients. In this case, a card is provided with the speed range and increment and this is followed by the bearing data. An input card is given for each coefficient at each bearing. Each card contains three items, namely the above mentioned three speed components. The sequence of the input cards is: first all the cards for the translatory motion and then all the cards for the rotational motion. The cards for the rotational motion are not required if word 6, card 3, support tilting, is zero. The cards should be given in the following order: $K_{xx}, \omega C_{xx}, K_{xy}, \omega C_{xy}, K_{yy}, \omega C_{yy}, K_{yx}, \omega C_{yx}$ for bearing 1, $K_{xx}, \omega C_{xx}$------ωC_{yx} for bearing 2, etc. to the last bearing, then (if word 6, card 3 $\neq$ 0), $M_{xx}, \omega D_{xx}, M_{xy}, \omega D_{xy}, M_{yy}, \omega D_{yy}, M_{yx}, \omega D_{yx}$

for bearing 1, etc. to and including the last bearing.

Card: (1P3E14.6) - Speed data.
Word 1 - initial speed.
Word 2 - final speed.
Word 3 - speed increment.

Card: (1P3E14.6) - Bearing data - in the order defined above with three values on each card as follows:
Word 1 - the coefficient A_0 of the expression $A = A_0 + A_1\omega + A_2\omega^2$
Word 2 - the coefficient A_1 of this expression.
Word 3 - the coefficient A_2 of this expression.

If word 8, card 3, is greater than 1, the program assumes the bearing data will be provided for each value of speed. In this case, a card is provided with a single speed value and this is followed by the bearing data as follows: all of the translatory stiffness and damping coefficients are provided in the order; two cards for each bearing. The first card contains the X coordinate translatory coefficients $K_{xx}, \omega C_{xx}, K_{xy}$ and ωC_{xy} and the second card the y coordinate translatory coefficients $K_{yy}, \omega C_{yy}, K_{yx}$ and ωC_{yx}, both cards for bearing one followed by two cards for bearing two, etc., to the total number of bearings. Again, if word 6, card 3 is zero, no rotational parameters are required, otherwise, they are provided in a similar manner: one card of values $M_{xx}, \omega D_{xx}, M_{xy}, \omega D_{xy}$ and a second card of $M_{yy}, \omega D_{yy}, M_{yx}, \omega D_{yx}$ for bearing 1 followed by two cards each for the remaining bearings.

The card format in this case is then:

Card: (1PE14.6) - Speed.
Card: (1P4E14.6) - Bearing data in the order defined above with these values on each card as follows:

Card 1 - Word 1 - $K_{xx,n}$
Word 2 - $\omega C_{xx,n}$
Word 3 - $K_{xy,n}$
Word 4 - $\omega C_{xy,n}$

Card 2 - Word 1 - $K_{yy,n}$

Word 2 $-\omega C_{yy,n}$

Word 3 - $K_{yx,n}$

Word 4 $-\omega C_{yx,n}$

COMPUTER OUTPUT

The computer output is largely self-explanatory and each output item is identified by a descriptive text. There sample cases are shown on Page A1 to A17 . The output first lists all the input data, i.e. the two heading cards, the control words, Youngs Modulus, the rotor data, the bearing stations, the unbalance data, the coupling stations, the pedestal data, the speed data and finally the bearing data. Thereafter follow the results of the calculations with one set of output for each specified rotor speed. First, the speed is given in RPM which may be followed by the intput bearing data if it is new for every speed. Next, a 9-column list gives the rotor amplitude and bending moment at each rotor station. Both the amplitude and the bending moment require four quantities for their description. Since each rotor station whirls in an elliptical orbit it is convenient to express the four quantities in terms of the dimensions of the ellipse. Then the four quantities become:

1. the major axis of the ellipse: a(i.e. the maximum amplitude or the maximum bending moment during one revolution.
2. the minor axis of the ellipse: b
3. the angle between the x-axis of the overall reference system and the major axis of the ellipse: β, degrees (in output identified by: ANGLE X-MAJOR)
4. the phase angle with respect to the cosine-component of the unbalance: α , degrees.

The amplitude is given in thousands of an inch (mils) and the bending moment is given in lbs.in.

The selected method of presentation is illustrated by Fig. 5 and is given in detail in the analysis by Eqs. (37) to (41). However, a general description will also be given here.

The presentation is based on two reference coordinate systems. The first reference system is the stationary x-y system fixed with respect to ground, and which has at each rotor station its origin in the center of the statically deflected rotor (i.e. the deflection due to gravity). The x-y-system is "communicated" to the rotor via the specified values of the bearing spring and damping coefficients

(K_{xx}, K_{xy}, ωC_{xx}, etc.) and the corresponding pedestal data. In other words, the directions of the x-axis and the y-axis are chosen when preparing the computer input and the choice reflects in the input values used for K_{xx}, K_{xy} etc. Then the elliptical rotor orbit is centered in the origin of the x-y-system (i.e. the steady state shaft center), it has a major axis a, a minor axis b, and the orientation of the ellipse is defined by the angle β between the x-axis and the major axis, measured in direction of rotor rotation. Note, that both a, b and β vary along the rotor. A negative value for b signifies backward whirl.

Thus a,b and β specify the dimensions and the orientation of the elliptical orbit but one more quantity is needed to specify the position of the moving shaft center on the ellipse at any given time. The phase angle α serves this purpose. Let the major axis be the x_1-axis and the minor axis the y_1-axis (see Fig.5), i.e. the x_1-y_1-system is obtained by rotating the x-y-system an angle β in the direction of rotor rotation. Then the instantaneous position of the shaft center is given by:

$$x_1 = a\cos(\omega t+\alpha)$$
$$y_1 = b\sin(\omega t+\alpha)$$

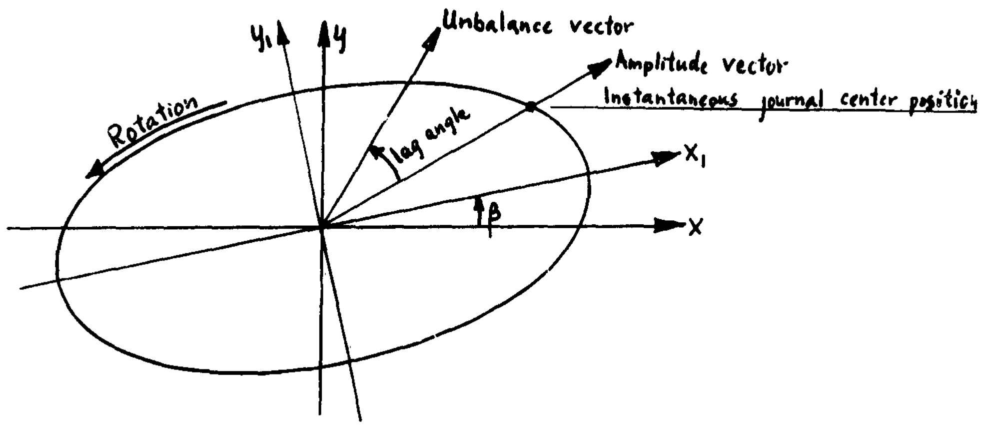
Note that the orientation of the x_1-y_1-system changes along the rotor since β does with respect to the x-y-system the instantaneous shaft center position is given by:

$$x = \sqrt{(a\cos\beta)^2+(b\sin\beta)^2}\cdot\cos\left(\omega t+\alpha+\tan^{-1}\left(\tfrac{b}{a}\tan\beta\right)\right)$$
$$y = \sqrt{(a\sin\beta)^2+(b\cos\beta)^2}\ \sin\left(\omega t+\alpha+\tan^{-1}\left(\tfrac{a}{b}\tan\beta\right)\right)$$

In addition to determining the instantaneous position of the shaft center with respect to a stationary coordinate system it also may be desired to know the position with respect to the rotor unbalance. The location of the unbalance in the rotor is defined by a coordinate system which is fixed in the rotating shaft and whose axes are called "the cosine axis" and "the sine axis". Hence, each unbalance consist of two components: a cosine component and a sine component (in the analysis the symbols U_x and U_y are used, respectively, see Eq.s(10) and (11)). The instantaneous orientation of the cosine-system is defined by the angle (ωt) between the fixed axis and the cosine-axis. Thus, the instantaneous phase angle between the amplitude vector (i.e. the radius vector from the center of the elliptical orbit going through the instantaneous shaft center position) and the total rotor unbalance vector is:

angle by which amplitude vector lags unbalance vector =

$$\omega t - \beta + \tan^{-1}\left(\frac{\Sigma U_{yn}}{\Sigma U_{xn}}\right) - \tan^{-1}\left(\frac{b}{a}\tan(\omega t + \alpha)\right)$$



Here, ΣU_{xn} and ΣU_{yn} indicates the summations of the cosine-components and the sine-components, respectively, of all unbalances. It is seen that the lag-angle is not constant as the shaft center moves around its orbit. It attains its maximum and minimum values when:

$$\tan^2(\omega t + \alpha) = \frac{a}{b}$$

Although the discussion above is primarily aimed at describing the motion of the shaft center (i.e. the computer output for the amplitude) the same description applies to the output for the bending moment. However, for each rotor station the output lists one line for the amplitude but two lines for the bending moment. Whereas the output for the amplitude applies at the rotor station itself the bending moment has one value immediately to the left of the station and another value immediately to the right of the station. The output gives the left hand value first (i.e. the output gives M_n and M_n' respectively, see Fig. 2). The two values are in general the same unless the particular station is a bearing station which resists tilting. The last listed value of the bending moment should always be zero (i.e. corresponding values of the major and minor axis should be zero). In general they are not exactly zero but very small. The amount by which the values differ from zero gives an indication of the accuracy of the calculation. Note, that for this reason the last values of the angles β and α are meaningless.

Following the output for the amplitude and for the bending moment come the results for the force transmitted to the bearing housing (equal to the dynamic bearing reaction). If the pedestals are flexible, the force transmitted to the foundation and the amplitude of the pedestal mass are also given. Each of the three quantities are presented in two ways: first in terms of the corresponding ellipse (i.e. in analogy to the rotor amplitude) and secondly by their x and y-components. Thus, if the transmitted force is F the output gives 8 quantities: the major axis, the minor axis, the orientation angle β, the phase angle α, $|F_x|$, α_x, $|F_y|$ and α_y where the last four items are defined by:

force in x-direction: $F_x = |F_x| \cos(\omega t + \alpha_x)$

force in y-direction: $F_y = |F_y| \sin(\omega t + \alpha_y)$

The transmitted force is given in lbs and the pedestal amplitude in thousands of an inch (mils).

The next line of output serves as a check on the calculation. It gives the energy per revolution put into the system by the unbalance forces and the energy dissipated per revolution in the bearings and pedestals. Theoretically, the two values should be equal but numerical inaccuracies in the calculations always cause discrepancy. Normally they differ in the fifth or sixth decimal place. The energy is given in lbs.inch/revolution.

To convert it into HP multiply the energy value by the speed in RPM and divide by $3.96 \cdot 10^5$.

If the input does not include any gyroscopic moment effects the calculations are repeated for a new rotor speed and the output will follow the description given above. If the gyroscopic moment is included there are two sets of output for each rotor speed, each set having the format as explained above. The first set applies to a rotor without any gyroscopic moments, and the second set gives the final result for the calculation with the gyroscopic moment included. The two sets are separated by a two column list giving the sequential results of the iterations needed to perform the gyroscopic moment calculation. The first column identifies the iteration number and the second column gives the relative convergence of the iteration procedure as explained in describing the computer input.

ACKNOWLEDGEMENT

The analysis and the computer program described in the present report is an adaptation of a basic computer program developed on an internal research program by Mechanical Technology Inc.

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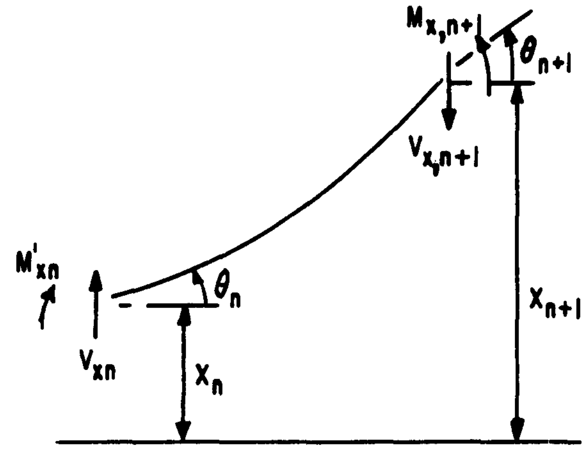

FIG. I SHAFT SECTION BETWEEN TWO MASS STATIONS

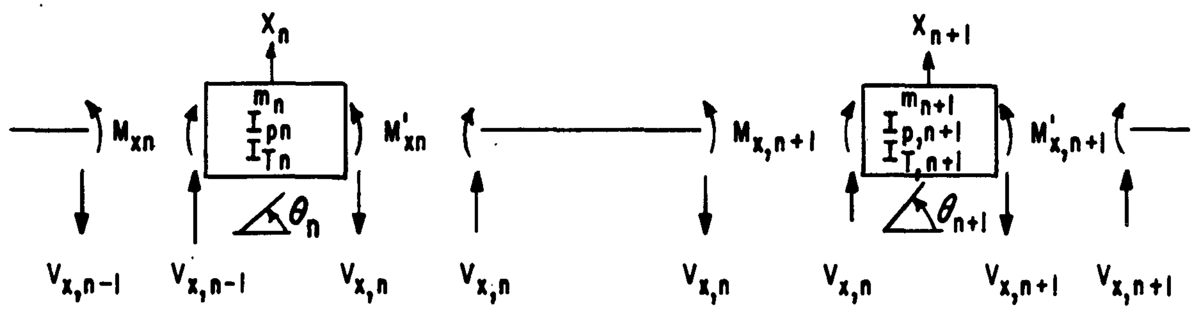

FIG. 2 CONVENTION AND NOMENCLATURE FOR ROTOR CALCULATION

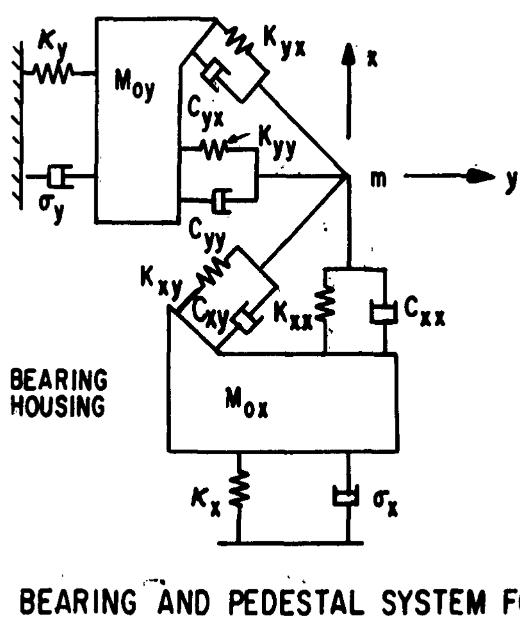

FIG. 3 BEARING AND PEDESTAL SYSTEM FOR TRANSLATORY MOTION

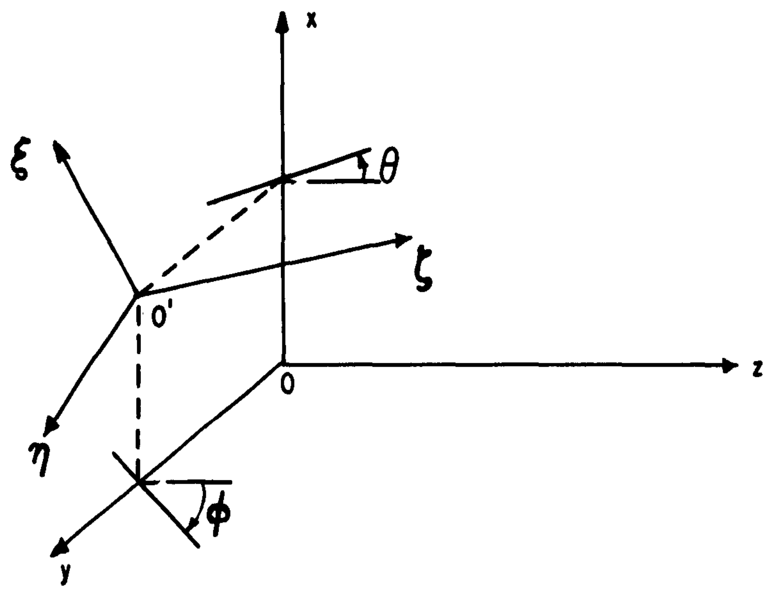

FIG. 4 GYROSCOPIC MOMENT CALCULATION

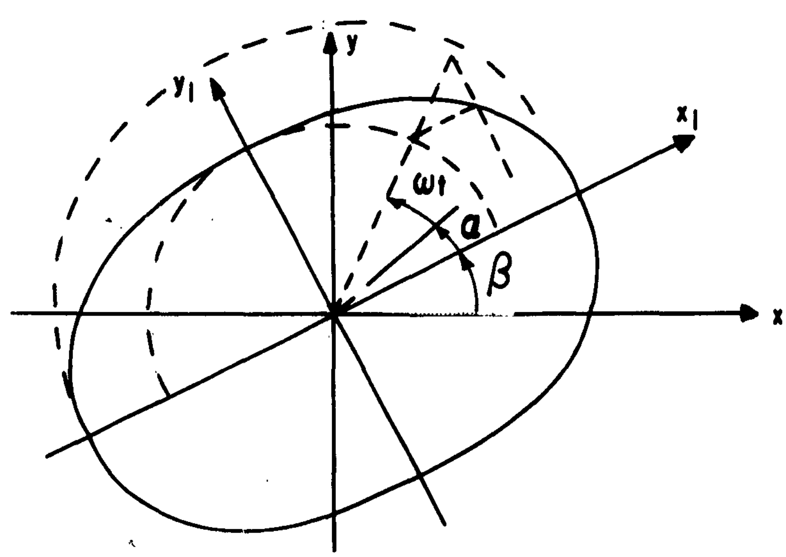

FIG. 5 ELLIPTICAL WHIRL PATH

NOMENCLATURE

A	Cross sectional area of shaft, in^2
a, b	Major and minor axis of ellipse, in (or: lbs, lbs.in)
a_n, b_n, c_n, d_n	Influence coefficients for shaft section, see Eqs.(4) to (7)
C	Radial bearing clearance, inch
$C_{xx}, C_{xy}, C_{yx}, C_{yy}$	Bearing damping coefficient for translatory whirl, lbs.sec/in
$D_{xx}, D_{xy}, D_{yx}, D_{yy}$	Bearing damping coefficients for conical whirl, lbs.in.sec/radian
E	Youngs modulus, $lbs.in^2$
e	Rotor mass eccentricity, inch
F_x, F_y	x-and y-components of bearing reaction, lbs.
I	Cross-sectional moment of inertia of shaft, (I_n between stations n and (n+1)), in^4
I_p	Polar mass moment of inertia of a rotor mass, $lbs.in.sec^2$ (in input: $lbs.in^2$)
I_T	Transverse mass moment of inertia of a rotor mass, $lbs.in.sec^2$ (in input: $lbs.in^2$)
$K_{xx}, K_{xy}, K_{yx}, K_{yy}$	Bearing spring coefficients for translatory whirl, lbs/in.
K_r	Rotor stiffness, lbs/in
L_n	Length of shaft section between station n and (n+1), inch
L	Bearing length, inch
l	Rotor length, inch
M	Bending moment (M_n to the left, M'_n to the right of station n), lbs.in.
M_x, M_y	x and y-components of pedestal mass, $lbs.sec^2/in$ (in input: lbs).
M_T	Total rotor mass, $lbs.sec^2/in$
$M_{xx}, M_{xy}, M_{yx}, M_{yy}$	Bearing spring coefficients for conical whirl, lbs.in/radian
m_n	Mass at rotor station n, $lbs.sec^2/in$ (in input: lbs)
P_x, P_y	x and y-components for force transmitted to base, lbs.

t	Time, seconds
U_x, U_y	Cosine and sine-components of unbalance, lbs.sec^2 (in input: oz.in)
V	Shear force (V_n to the right of station n), lbs.
W	Bearing reaction, lbs.
x, y	Components of the rotor amplitude, inch (in output: mils)
z	Coordinate along length of rotor, inch.
α	Phase angle between amplitude radius vector and unbalance see Fig. 5, radians
β	Angle between major axis and x-axis, see Fig. 5, radians
δ_x, δ_y	Pedestal damping coefficients for conical whirl, lbs.in.sec/rad.
θ, φ	$= \frac{dx}{dz}, \frac{dy}{dz}$, components of the slope of the deflected rotor, rad.
$\varkappa_x, \varkappa_y$	Pedestal spring coefficients, lbs/in.
σ_x, σ_y	Pedestal damping coefficients, lbs.sec/in.
ω	Angular speed of rotor, radians/sec.
ω_n	Critical rotor speed, radians/sec.

Subscripts and Superscripts

x	in x-direction
y	in y-direction
xx, xy, yx, yy	first index gives force direction, second index gives amplitude direction.
c	cosine component
s	sine component
n	applies to station n
p	pedestal
$(\ )'$	relative between journal and pedestal

APPENDIX

COMPARISON NEVILLE-P.W.,150 DEGR.ARC,E=.1,C/DELTA=10

10-23-63

```
  15    2    1    0    0    0    0    1    0    0
 3.2156250E+07  1.0000000E+00
 3.189466E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 6.378931E+00  3.000000E+00  6.000000E-02
 3.189466E+00  0.000000E+00  0.000000E+00
   1   15
   8  1.4288860E+00  0.0000000E+00
 1.100000E+03  4.200000E+03  1.000000E+03
 4.051000E+02  0.000000E+00  0.000000E+00
 5.711000E+02  0.000000E+00  0.000000E+00
-2.418000E+02  0.000000E+00  0.000000E+00
 6.876000E+02  0.000000E+00  0.000000E+00
-1.793000E+02  0.000000E+00  0.000000E+00
 1.878000E+03  0.000000E+00  0.000000E+00
 9.651000E+02  0.000000E+00  0.000000E+00
 6.876000E+02  0.000000E+00  0.000000E+00
 4.051000E+02  0.000000E+00  0.000000E+00
 5.711000E+02  0.000000E+00  0.000000E+00
-2.418000E+02  0.000000E+00  0.000000E+00
 6.876000E+02  0.000000E+00  0.000000E+00
-1.793000E+02  0.000000E+00  0.000000E+00
 1.878000E+03  0.000000E+00  0.000000E+00
 9.651000E+02  0.000000E+00  0.000000E+00
 6.876000E+02  0.000000E+00  0.000000E+00
```

UNBALANCE RESPONSE OF ROTOR WITH NON-UNIFORM BEARING SUPPORTS PN0011
MECHANICAL TECHNOLOGY,INC.

COMPARISON NEVILLE-P.W.,150 DEGR.ARC,E=.1,C/DELTA=10
10-23-63

STATIONS	NO.BRGS.	NO.UNBAL.	NO.COUPL.	PED.FLEX.	BRG.MOMENT	GYRO.MOM.	NO.CASES	DIAGNOSTIC	INPUT
15	2	1	0	0	0	0	1	0	0

YOUNGS MODULUS	SCALE FACTOR
3.2156250E 07	1.0000000E 00

ROTOR DATA

STATION NO.	MASS	LENGTH	CROSS SECT.INERTIA
1	3.1894660E 00	3.0000000E 00	5.9999999E-02
2	6.3789310E 00	3.0000000E 00	5.9999999E-02
3	6.3789310E 00	3.0000000E 00	5.9999999E-02
4	6.3789310E 00	3.0000000E 00	5.9999999E-02
5	6.3789310E 00	3.0000000E 00	5.9999999E-02
6	6.3789310E 00	3.0000000E 00	5.9999999E-02
7	6.3789310E 00	3.0000000E 00	5.9999999E-02
8	6.3789310E 00	3.0000000E 00	5.9999999E-02
9	6.3789310E 00	3.0000000E 00	5.9999999E-02
10	6.3789310E 00	3.0000000E 00	5.9999999E-02
11	6.3789310E 00	3.0000000E 00	5.9999999E-02
12	6.3789310E 00	3.0000000E 00	5.9999999E-02
13	6.3789310E 00	3.0000000E 00	5.9999999E-02
14	6.3789310E 00	3.0000000E 00	5.9999999E-02
15	3.1894660E 00	0.	0.

BEARING STATIONS
1 15

UNBALANCE ST.	X-UNBALANCE	Y-UNBALANCE
8	1.4288860E 00	0.

INITIAL SPEED	FINAL SPEED	SPEED INCR.
1.100000E 03	4.200000E 03	1.000000E 03

BEARING AT STATION NO. 1

KXX	CXX	KXY	CXY	KYY	CYY	KYX	CYX
4.051000E 02	5.711000E 02	-2.418000E 02	6.876000E 02	-1.793000E 02	1.878000E 03	9.651000E 02	6.876000E 02
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

BEARING AT STATION NO. 15

KXX	CXX	KXY	CXY	KYY	CYY	KYX	CYX
4.051000E 02	5.711000E 02	-2.418000E 02	6.876000E 02	-1.793000E 02	1.878000E 03	9.651000E 02	6.876000E 02
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

ROTOR SPEED= 1.1000000E 03RPM

	AMPLITUDE				BENDING MOMENT			
STATION	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE
1	1.98886E 00	1.20160E 00	-6.38749E 01	4.66800E 01	0.	0.	0.	0.
					0.	0.	0.	0.
2	1.90430E 00	1.41522E 00	3.98059E 01	-2.96392E 01	1.53171E 01	1.90121E 00	6.92136E 01	-2.31471E-01
					1.53171E 01	1.90121E 00	6.92136E 01	-2.31471E-01
3	2.85987E 00	1.15677E 00	4.99872E 01	-1.94579E 01	2.99343E 01	3.42416E 00	7.08185E 01	1.37347E 00
					2.99343E 01	3.42416E 00	7.08185E 01	1.37347E 00
4	3.85750E 00	1.06319E 00	5.61573E 01	-1.32878E 01	4.30570E 01	4.57274E 00	7.24556E 01	3.01053E 00
					4.30570E 01	4.57274E 00	7.24556E 01	3.01053E 00
5	4.74386E 00	1.03641E 00	5.98285E 01	-9.61654E 00	5.39807E 01	5.36031E 00	7.42182E 01	4.77315E 00

					5.39807E 01	5.36031E 00	7.42182E 01	4.77315E 00
6	5.42763E 00	1.03195E 00	6.20287E 01	-7.41640E 00	6.21475E 01	5.82946E 00	7.62255E 01	6.78041E 00
					6.21475E 01	5.82946E 00	7.62255E 01	6.78041E 00
7	5.85498E 00	1.03153E 00	6.32466E 01	-6.19842E 00	6.72031E 01	6.07852E 00	7.86422E 01	9.19717E 00
					6.72031E 01	6.07852E 00	7.86422E 01	9.19717E 00
8	5.99920E 00	1.03065E 00	6.36524E 01	-5.79267E 00	6.90631E 01	6.30461E 00	8.17066E 01	1.22615E 01
					6.90631E 01	6.30461E 00	8.17066E 01	1.22615E 01
9	5.85498E 00	1.03153E 00	6.32466E 01	-6.19841E 00	6.72031E 01	6.07852E 00	7.86422E 01	9.19717E 00
					6.72031E 01	6.07852E 00	7.86422E 01	9.19717E 00
10	5.42763E 00	1.03195E 00	6.20287E 01	-7.41640E 00	6.21475E 01	5.82946E 00	7.62255E 01	6.78041E 00
					6.21475E 01	5.82946E 00	7.62255E 01	6.78041E 00
11	4.74386E 00	1.03641E 00	5.98285E 01	-9.61653E 00	5.39807E 01	5.36031E 00	7.42182E 01	4.77315E 00
					5.39807E 01	5.36031E 00	7.42182E 01	4.77315E 00
12	3.85750E 00	1.06319E 00	5.61573E 01	-1.32878E 01	4.30570E 01	4.57275E 00	7.24556E 01	3.01053E 00
					4.30570E 01	4.57275E 00	7.24556E 01	3.01053E 00
13	2.85987E 00	1.15678E 00	4.99872E 01	-1.94579E 01	2.99343E 01	3.42416E 00	7.08185E 01	1.37347E 00
					2.99343E 01	3.42416E 00	7.08185E 01	1.37347E 00
14	1.90430E 00	1.41522E 00	3.98059E 01	-2.96392E 01	1.53171E 01	1.90121E 00	6.92136E 01	-2.31489E-01
					1.53171E 01	1.90121E 00	6.92136E 01	-2.31489E-01
15	1.98886E 00	1.20160E 00	-6.38749E 01	4.66800E 01	6.34228E-06	1.43402E-07	-7.43236E 01	5.17038E 01
					6.34228E-06	1.43402E-07	-7.43236E 01	5.17038E 01

FORCE TRANSMITTED TO BEARING HOUSING

BRG.NO.	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	X-AMPLITUDE	X-PHASE ANG	Y-AMPLITUDE	Y-PHASE ANG
1	5.07950E 00	6.93823E-01	6.74809E 01	-1.96413E 00	2.04826E 00	-1.63729E 02	4.69971E 00	-9.52054E 01
15	5.07949E 00	6.93824E-01	6.74809E 01	-1.96412E 00	2.04826E 00	-1.63729E 02	4.69971E 00	-9.52054E 01

ENERGY INPUT= 5.7399159E-02 ENERGY DISSIPATED= 5.7399139E-02

ROTOR SPEED= 2.1000000E 03RPM

	AMPLITUDE				BENDING MOMENT			
STATION	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE
1	1.06991E 01	3.65548E 00	-3.50981E 01	-2.06072E 01	0.	0.	0.	0.
					0.	0.	0.	0.
2	7.38227E 00	1.97104E 00	-2.94873E 01	-1.49963E 01	2.19300E 01	3.58031E 00	-8.00937E 01	-6.56028E 01
					2.19300E 01	3.58031E 00	-8.00937E 01	-6.56028E 01
3	4.30089E 00	1.87536E-01	-2.03724E 01	-5.88142E 00	5.49554E 01	1.84867E 01	-6.98835E 01	-5.53926E 01
					5.49554E 01	1.84867E 01	-6.98835E 01	-5.53926E 01
4	1.72578E 00	1.70028E 00	8.28416E 01	-8.26688E 01	9.43402E 01	3.77932E 01	-6.15910E 01	-4.71001E 01
					9.43402E 01	3.77932E 01	-6.15910E 01	-4.71001E 01
5	3.69692E 00	2.26188E-01	-8.36093E 01	-6.91184E 01	1.34205E 02	5.65590E 01	-5.48200E 01	-4.03291E 01
					1.34205E 02	5.65590E 01	-5.48200E 01	-4.03291E 01
6	5.42441E 00	1.51377E 00	-7.42503E 01	-5.97594E 01	1.68982E 02	7.10076E 01	-4.88517E 01	-3.43608E 01
					1.68982E 02	7.10076E 01	-4.88517E 01	-3.43608E 01
7	6.58326E 00	2.26342E 00	-6.91863E 01	-5.46954E 01	1.94500E 02	7.80831E 01	-4.30731E 01	-2.85821E 01
					1.94500E 02	7.80831E 01	-4.30731E 01	-2.85821E 01
8	6.98651E 00	2.51364E 00	-6.75302E 01	-5.30393E 01	2.08702E 02	7.52680E 01	-3.70573E 01	-2.25664E 01
					2.08702E 02	7.52680E 01	-3.70573E 01	-2.25664E 01
9	6.58326E 00	2.26342E 00	-6.91863E 01	-5.46954E 01	1.94500E 02	7.80831E 01	-4.30731E 01	-2.85821E 01
					1.94500E 02	7.80831E 01	-4.30731E 01	-2.85821E 01
10	5.42440E 00	1.51377E 00	-7.42503E 01	-5.97594E 01	1.68982E 02	7.10076E 01	-4.88517E 01	-3.43608E 01
					1.68982E 02	7.10076E 01	-4.88517E 01	-3.43608E 01
11	3.69692E 00	2.26190E-01	-8.36093E 01	-6.91183E 01	1.34205E 02	5.65590E 01	-5.48200E 01	-4.03291E 01
					1.34205E 02	5.65590E 01	-5.48200E 01	-4.03291E 01
12	1.72578E 00	1.70028E 00	8.28400E 01	-8.26671E 01	9.43403E 01	3.77933E 01	-6.15910E 01	-4.71000E 01
					9.43403E 01	3.77933E 01	-6.15910E 01	-4.71000E 01
13	4.30089E 00	1.87534E-01	-2.03724E 01	-5.88141E 00	5.49555E 01	1.84868E 01	-6.98835E 01	-5.53925E 01
					5.49555E 01	1.84868E 01	-6.98835E 01	-5.53925E 01
14	7.38227E 00	1.97104E 00	-2.94872E 01	-1.49963E 01	2.19300E 01	3.58039E 00	-8.00936E 01	-6.56126E 01
					2.19300E 01	3.58039E 00	-8.00936E 01	-6.56126E 01
15	1.06991E 01	3.65548E 00	-3.50981E 01	-2.06072E 01	1.06754E-04	5.86145E-05	-3.17994E 01	-2.45105E 01
					1.06754E-04	5.86145E-05	-3.17994E 01	-2.45105E 01

FORCE TRANSMITTED TO BEARING HOUSING

BRG.NO.	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	X-AMPLITUDE	X-PHASE ANG	Y-AMPLITUDE	Y-PHASE ANG
1	4.75041E 00	1.98208E 00	8.75523E 01	-7.79569E 01	1.99063E 00	-1.73806E 02	4.74683E 00	-1.68979E 02
15	4.75041E 00	1.98207E 00	8.75523E 01	-7.79568E 01	1.99063E 00	-1.73806E 02	4.74683E 00	-1.68979E 02

ENERGY INPUT= 1.3534893E-01   ENERGY DISSIPATED= 1.3534909E-01

ROTOR SPEED= 3.1000000E 03RPM

	AMPLITUDE				BENDING MOMENT			
STATION	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE
1	4.65257E 00	2.43945E-02	1.00937E 01	3.39380E 01	0.	0.	0.	0.
					0.	0.	0.	0.
2	2.98194E 00	3.65708E-01	2.00748E 01	4.39192E 01	1.83017E 01	1.10044E 01	7.30416E 01	-8.31142E 01
					1.83017E 01	1.10044E 01	7.30416E 01	-8.31142E 01
3	1.87915E 00	1.67531E-01	4.95141E 01	7.33584E 01	3.33131E 01	3.04203E 01	-1.10088E 01	1.28357E 01
					3.33131E 01	3.04203E 01	-1.10088E 01	1.28357E 01
4	1.76619E 00	9.98669E-01	7.24561E 01	-8.36996E 01	5.94728E 01	3.56984E 01	-6.86948E 00	1.69749E 01
					5.94728E 01	3.56984E 01	-6.86948E 00	1.69749E 01
5	2.23786E 00	1.84023E 00	-9.95872E 00	1.38857E 01	8.16990E 01	3.30338E 01	-3.25570E 00	2.05886E 01
					8.16990E 01	3.30338E 01	-3.25570E 00	2.05886E 01
6	3.17846E 00	1.84108E 00	-6.52924E 00	1.73151E 01	9.35255E 01	2.20466E 01	9.51690E-01	2.47960E 01
					9.35255E 01	2.20466E 01	9.51690E-01	2.47960E 01
7	3.72494E 00	1.77581E 00	-4.71648E 00	1.91279E 01	9.12900E 01	3.98346E 00	8.11006E 00	3.19544E 01
					9.12900E 01	3.98346E 00	8.11006E 00	3.19544E 01
8	3.88833E 00	1.72679E 00	-4.07864E 00	1.97657E 01	7.98519E 01	1.31003E 01	2.70678E 01	5.09121E 01
					7.98519E 01	1.31003E 01	2.70678E 01	5.09121E 01
9	3.72494E 00	1.77581E 00	-4.71648E 00	1.91279E 01	9.12900E 01	3.98349E 00	8.11006E 00	3.19544E 01
					9.12900E 01	3.98349E 00	8.11006E 00	3.19544E 01
10	3.17846E 00	1.84109E 00	-6.52923E 00	1.73151E 01	9.35256E 01	2.20467E 01	9.51695E-01	2.47960E 01
					9.35256E 01	2.20467E 01	9.51695E-01	2.47960E 01
11	2.23786E 00	1.84023E 00	-9.95868E 00	1.38857E 01	8.16991E 01	3.30339E 01	-3.25567E 00	2.05886E 01
					8.16991E 01	3.30339E 01	-3.25567E 00	2.05886E 01
12	1.76619E 00	9.98669E-01	7.24561E 01	-8.36996E 01	5.94729E 01	3.56985E 01	-6.86942E 00	1.69748E 01
					5.94729E 01	3.56985E 01	-6.86942E 00	1.69748E 01
13	1.87915E 00	1.67532E-01	4.95141E 01	7.33584E 01	3.33133E 01	3.04204E 01	-1.10079E 01	1.28349E 01
					3.33133E 01	3.04204E 01	-1.10079E 01	1.28349E 01
14	2.98194E 00	3.65711E-01	2.00748E 01	4.39191E 01	1.83017E 01	1.10046E 01	7.30412E 01	-8.31137E 01
					1.83017E 01	1.10046E 01	7.30412E 01	-8.31137E 01
15	4.65258E 00	2.43896E-02	1.00937E 01	3.39380E 01	2.58734E-04	6.82225E-05	3.48030E-01	-2.78847E 01
					2.58734E-04	6.82225E-05	3.48030E-01	-2.78847E 01

FORCE TRANSMITTED TO BEARING HOUSING

BRG.NO.	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	X-AMPLITUDE	X-PHASE ANG	Y-AMPLITUDE	Y-PHASE ANG
1	7.27333E 00	8.11468E-01	6.10019E 01	8.48462E 01	3.59669E 00	9.62272E 01	6.37367E 00	1.71308E 02
15	7.27333E 00	8.11465E-01	6.10019E 01	8.48462E 01	3.59669E 00	9.62271E 01	6.37367E 00	1.71308E 02

ENERGY INPUT= 1.1627376E-01   ENERGY DISSIPATED= 1.1627366E-01

ROTOR SPEED= 4.1000000E 03RPM

	AMPLITUDE				BENDING MOMENT			
STATION	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE
1	3.27710E 00	1.40354E 00	1.79322E 01	8.54518E 00	0.	0.	0.	0.
					0.	0.	0.	0.
2	1.90436E 00	4.54832E-01	2.62861E 01	1.68991E 01	2.28648E 01	1.12893E 01	8.36040E 01	7.42172E 01
					2.28648E 01	1.12893E 01	8.36040E 01	7.42172E 01
3	1.11208E 00	1.30279E-01	6.38583E 01	5.44714E 01	4.48226E 01	3.49168E 01	-8.92834E 01	8.13299E 01
					4.48226E 01	3.49168E 01	-8.92834E 01	8.13299E 01
4	1.53226E 00	1.05743E 00	8.88012E 01	7.94144E 01	6.08343E 01	6.00992E 01	4.88992E 00	-4.50201E 00
					6.08343E 01	6.00992E 01	4.88992E 00	-4.50201E 00
5	1.97999E 00	1.95466E 00	4.90061E 00	-4.48982E 00	7.82656E 01	6.25509E 01	8.26379E 00	-1.12341E 00
					7.82656E 01	6.25509E 01	8.26379E 00	-1.12341E 00
6	2.59471E 00	2.14234E 00	7.74648E 00	-1.64065E 00	7.88194E 01	4.83564E 01	1.24305E 01	3.04348E 00
					7.88194E 01	4.83564E 01	1.24305E 01	3.04348E 00

```
SAMPLE CALCULATION NO.1
ROTOR IN RIGID PEDESTALS,NO GYROSCOPIC MOMENT
  27    4    1     0     0     0     0     1     0     0
 3.1300000E+07  1.0000000E+00
 7.600000E+01  3.750000E+00  5.250000E+01
 2.140000E+01  4.090000E+00  4.880000E+01
 3.000000E+01  5.310000E+00  4.880000E+01
 1.250000E+02  6.620000E+00  1.440000E+02
 1.240000E+02  2.250000E+00  3.020000E+02
 3.820000E+02  5.910000E+00  7.788000E+02
 8.760000E+02  9.780000E+00  2.580000E+03
 7.770000E+02  7.190000E+00  8.380000E+02
 5.700000E+02  8.570000E+00  3.950000E+02
 4.530000E+02  7.190000E+00  3.850000E+02
 2.350000E+02  5.000000E+00  3.210000E+02
 3.000000E+02  6.620000E+00  6.130000E+02
 3.130000E+02  4.870000E+00  1.930000E+02
 8.200000E+01  2.130000E+00  6.800000E+01
 3.200000E+01  3.190000E+00  4.180000E+01
 3.672000E+01  7.420000E+00  2.000000E+01
 2.821000E+01  8.650000E+00  5.476000E+00
 9.043000E+01  1.237500E+01  3.120000E-01
 4.297000E+00  1.237500E+00  3.120000E-01
 4.297000E+00  1.237500E+00  3.120000E-01
 7.852000E+01  4.750000E+00  2.000000E+01
 3.423000E+01  5.320000E+00  1.092000E+02
 9.820000E+01  6.190000E+00  8.540000E+02
 2.431000E+02  6.510000E+00  1.602000E+03
 3.300000E+02  6.210000E+00  8.360000E+02
 2.843000E+02  4.330000E+00  1.554000E+02
 6.370000E+01  0.000000E+00  0.000000E+00
   3   16   22   27
  10  1.0000000E+01  0.0000000E+00
 1.000000E+03  5.100000E+03  2.000000E+03
 1.542000E+06  0.000000E+00  0.000000E+00
 3.362000E+06  0.000000E+00  0.000000E+00
-8.010000E+05  0.000000E+00  0.000000E+00
 1.302000E+06  0.000000E+00  0.000000E+00
 1.542000E+05  0.000000E+00  0.000000E+00
 1.040000E+06  0.000000E+00  0.000000E+00
 9.720000E+05  0.000000E+00  0.000000E+00
 1.302000E+06  0.000000E+00  0.000000E+00
 1.542000E+06  0.000000E+00  0.000000E+00
 3.362000E+06  0.000000E+00  0.000000E+00
-8.010000E+05  0.000000E+00  0.000000E+00
 1.302000E+06  0.000000E+00  0.000000E+00
 1.542000E+05  0.000000E+00  0.000000E+00
 1.040000E+06  0.000000E+00  0.000000E+00
 9.720000E+05  0.000000E+00  0.000000E+00
 1.302000E+06  0.000000E+00  0.000000E+00
 1.463000E+06  0.000000E+00  0.000000E+00
 3.296000E+06  0.000000E+00  0.000000E+00
-1.045000E+06  0.000000E+00  0.000000E+00
 1.781000E+06  0.000000E+00  0.000000E+00
-1.197000E+05  0.000000E+00  0.000000E+00
 1.754000E+06  0.000000E+00  0.000000E+00
 1.175000E+06  0.000000E+00  0.000000E+00
 1.781000E+06  0.000000E+00  0.000000E+00
 1.463000E+06  0.000000E+00  0.000000E+00
 3.296000E+06  0.000000E+00  0.000000E+00
-1.045000E+06  0.000000E+00  0.000000E+00
```

1.781000E+06	0.000000E+00	0.000000E+00
-1.197000E+05	0.000000E+00	0.000000E+00
1.754000E+06	0.000000E+00	0.000000E+00
1.175000E+06	0.000000E+00	0.000000E+00
1.781000E+06	0.000000E+00	0.000000E+00

UNBALANCE RESPONSE OF ROTOR WITH NON-UNIFORM BEARING SUPPORTS PN0011
MECHANICAL TECHNOLOGY, INC.

SAMPLE CALCULATION NO.1
ROTOR IN RIGID PEDESTALS,NO GYROSCOPIC MOMENT

STATIONS	NO.BRGS.	NO.UNBAL.	NO.COUPL.	PED.FLEX.	BRG.MOMENT	GYRO.MOM.	NO.CASES	DIAGNOSTIC	INPUT
27	4	1	0	0	0	0	1	0	0

YOUNGS MODULUS	SCALE FACTOR
3.1300000E 07	1.0000000E 00

ROTOR DATA

STATION NO.	MASS	LENGTH	CROSS SECT.INERTIA
1	7.6000000E 01	3.7500000E 00	5.2500000E 01
2	2.1400000E 01	4.0900000E 00	4.8799999E 01
3	3.0000000E 01	5.3099999E 00	4.8799999E 01
4	1.2500000E 02	6.6199999E 00	1.4400000E 02
5	1.2400000E 02	2.2500000E 00	3.0200000E 02
6	3.8200000E 02	5.9100000E 00	7.7879999E 02
7	8.7600000E 02	9.7800000E 00	2.5800000E 03
8	7.7700000E 02	7.1900000E 00	8.3800000E 02
9	5.7000000E 02	8.5699999E 00	3.9500000E 02
10	4.5300000E 02	7.1900000E 00	3.8500000E 02
11	2.3500000E 02	5.0000000E 00	3.2100000E 02
12	3.0000000E 02	6.6199999E 00	6.1300000E 02
13	3.1300000E 02	4.8699999E 00	1.9300000E 02
14	8.1999999E 01	2.1300000E 00	6.8000000E 01
15	3.2000000E 01	3.1900000E 00	4.1799999E 01
16	3.6720000E 01	7.4200000E 00	2.0000000E 01
17	2.8210000E 01	8.6500000E 00	5.4760000E 00
18	9.0429999E 01	1.2375000E 01	3.1200000E-01
19	4.2970000E 00	1.2375000E 00	3.1200000E-01
20	4.2970000E 00	1.2375000E 00	3.1200000E-01
21	7.8519999E 01	4.7500000E 00	2.0000000E 01
22	3.4229999E 01	5.3200000E 00	1.0920000E 02
23	9.8199999E 01	6.1900000E 00	8.5400000E 02
24	2.4310000E 02	6.5100000E 00	1.6020000E 03
25	3.3000000E 02	6.2100000E 00	8.3599999E 02
26	2.8430000E 02	4.3300000E 00	1.5540000E 02
27	6.3699999E 01	0.	0.

BEARING STATIONS
3 16 22 27

UNBALANCE ST.	X-UNBALANCE	Y-UNBALANCE
10	1.0000000E 01	0.

INITIAL SPEED	FINAL SPEED	SPEED INCR.
1.000000E 03	5.100000E 03	2.000000E 03

BEARING AT STATION NO. 3

KXX	CXX	KXY	CXY	KYY	CYY	KYX	CYX
1.542000E 06	3.362000E 06	-8.010000E 05	1.302000E 06	1.542000E 05	1.040000E 06	9.720000E 05	1.302000E 06
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

BEARING AT STATION NO. 16

KXX	CXX	KXY	CXY	KYY	CYY	KYX	CYX
1.542000E 06	3.362000E 06	-8.010000E 05	1.302000E 06	1.542000E 05	1.040000E 06	9.720000E 05	1.302000E 06
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

BEARING AT STATION NO. 22

KXX	CXX	KXY	CXY	KYY	CYY	KYX	CYX
1.463000E 06	3.296000E 06	-1.045000E 06	1.781000E 06	-1.197000E 05	1.754000E 06	1.175000E 06	1.781000E 06
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

BEARING AT STATION NO. 27

KXX	CXX	KXY	CXY	KYY	CYY	KYX	CYX
1.463000E 06	3.296000E 06	-1.045000E 06	1.781000E 06	-1.197000E 05	1.754000E 06	1.175000E 06	1.781000E 06
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

ROTOR SPEED= 1.0000000E 03RPM

	AMPLITUDE				BENDING MOMENT			
STATION	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE
1	1.39031E-02	3.56151E-03	8.83057E 01	-6.00205E 00	0.	0.	0.	0.
					0.	0.	0.	0.
2	1.44878E-02	3.64260E-03	-8.24257E 01	3.85309E 00	1.12551E-01	2.88318E-02	8.83057E 01	-6.00205E 00
					1.12551E-01	2.88318E-02	8.83057E 01	-6.00205E 00
3	1.57166E-02	3.14358E-03	-7.41769E 01	1.26943E 01	2.70677E-01	6.99795E-02	8.95281E 01	-4.70195E 00
					2.70677E-01	6.99795E-02	8.95281E 01	-4.70195E 00
4	1.77964E-02	1.99192E-03	-6.73328E 01	2.02372E 01	4.44206E 01	3.59125E 01	-4.72696E 01	4.37484E 01
					4.44206E 01	3.59125E 01	-4.72696E 01	4.37484E 01
5	2.02743E-02	5.52445E-04	-6.32120E 01	2.51265E 01	9.96411E 01	8.05672E 01	-4.72523E 01	4.37330E 01
					9.96411E 01	8.05672E 01	-4.72523E 01	4.37330E 01
6	2.10189E-02	1.30538E-04	-6.24072E 01	2.61719E 01	1.18263E 02	9.57625E 01	-4.72479E 01	4.37526E 01
					1.18263E 02	9.57625E 01	-4.72479E 01	4.37526E 01
7	2.28631E-02	8.97336E-04	-6.08939E 01	2.82804E 01	1.65935E 02	1.35790E 02	-4.72274E 01	4.39073E 01
					1.65935E 02	1.35790E 02	-4.72274E 01	4.39073E 01
8	2.57279E-02	2.45859E-03	-5.93821E 01	3.06830E 01	2.39642E 02	2.02188E 02	-4.71804E 01	4.43675E 01
					2.39642E 02	2.02188E 02	-4.71804E 01	4.43675E 01
9	2.75487E-02	3.37054E-03	-5.88326E 01	3.18242E 01	2.89984E 02	2.50846E 02	-4.71598E 01	4.47619E 01
					2.89984E 02	2.50846E 02	-4.71598E 01	4.47619E 01
10	2.85959E-02	3.49154E-03	-5.91346E 01	3.21671E 01	3.46381E 02	3.08588E 02	-4.71917E 01	4.52838E 01
					3.46381E 02	3.08588E 02	-4.71917E 01	4.52838E 01
11	2.81185E-02	2.40866E-03	-6.04862E 01	3.13166E 01	2.63633E 02	2.29287E 02	-4.76916E 01	4.53928E 01
					2.63633E 02	2.29287E 02	-4.76916E 01	4.53928E 01
12	2.71515E-02	1.11380E-03	-6.21557E 01	2.99786E 01	2.05217E 02	1.74123E 02	-4.81664E 01	4.54679E 01
					2.05217E 02	1.74123E 02	-4.81664E 01	4.54679E 01
13	2.54655E-02	9.21293E-04	-6.54590E 01	2.70980E 01	1.26473E 02	1.01146E 02	-4.91780E 01	4.55309E 01
					1.26473E 02	1.01146E 02	-4.91780E 01	4.55309E 01
14	2.40796E-02	2.49587E-03	-6.90474E 01	2.38083E 01	6.75969E 01	4.76444E 01	-5.08537E 01	4.52102E 01
					6.75969E 01	4.76444E 01	-5.08537E 01	4.52102E 01
15	2.34274E-02	3.19173E-03	-7.11462E 01	2.18345E 01	4.18154E 01	2.43401E 01	-5.27682E 01	4.41788E 01
					4.18154E 01	2.43401E 01	-5.27682E 01	4.41788E 01
16	2.24164E-02	4.17647E-03	-7.51413E 01	1.80118E 01	1.25391E 01	1.16061E 00	6.17133E 01	-1.93909E 01
					1.25391E 01	1.16061E 00	6.17133E 01	-1.93909E 01
17	2.04699E-02	5.38902E-03	-8.70794E 01	6.34127E 00	8.75040E 00	7.71164E-01	6.15986E 01	-1.96815E 01
					8.75040E 00	7.71164E-01	6.15986E 01	-1.96815E 01
18	1.83584E-02	4.38452E-03	7.82153E 01	-8.52345E 00	4.21843E 00	2.53877E-01	6.04918E 01	-2.12032E 01
					4.21843E 00	2.53877E-01	6.04918E 01	-2.12032E 01
19	2.17464E-03	3.84311E-04	8.71283E 01	-3.40218E 01	2.87382E 00	5.97280E-01	6.97978E 01	-1.13168E 01
					2.87382E 00	5.97280E-01	6.97978E 01	-1.13168E 01
20	1.39276E-03	3.89692E-04	-8.24983E 01	-4.58167E 01	3.57404E 00	6.91922E-01	6.85292E 01	-1.26470E 01
					3.57404E 00	6.91922E-01	6.85292E 01	-1.26470E 01
21	1.11001E-03	4.43961E-04	-7.60317E 01	-5.33104E 01	4.27610E 00	7.84611E-01	6.77061E 01	-1.35151E 01
					4.27610E 00	7.84611E-01	6.77061E 01	-1.35151E 01
22	8.50178E-04	4.12462E-04	-7.21069E 01	-5.59802E 01	6.98310E 00	1.13208E 00	6.62121E 01	-1.52189E 01
					6.98310E 00	1.13208E 00	6.62121E 01	-1.52189E 01
23	6.35236E-04	3.26221E-04	-7.06165E 01	-5.61340E 01	5.76372E 00	9.29153E-01	6.52204E 01	-1.58399E 01
					5.76372E 00	9.29153E-01	6.52204E 01	-1.58399E 01
24	3.99213E-04	2.10614E-04	-6.92457E 01	-5.49867E 01	4.37478E 00	6.20832E-01	6.29126E 01	-1.66583E 01
					4.37478E 00	6.20832E-01	6.29126E 01	-1.66583E 01

25	1.54105E-04	8.43193E-05	-6.13395E 01	-4.86896E 01	2.89481E 00	4.28728E-01	5.90760E 01	-1.71899E 01
					2.89481E 00	4.28728E-01	5.90760E 01	-1.71899E 01
26	8.69010E-05	2.96826E-05	-8.62112E 01	-7.17636E 01	1.55775E 00	1.34412E-01	5.53737E 01	-2.91954E 01
					1.55775E 00	1.34412E-01	5.53737E 01	-2.91954E 01
27	2.55734E-04	1.32739E-04	-7.47611E 01	-6.09850E 01	6.27897E-01	2.52735E-01	5.00218E 01	-7.13512E 01
					6.27897E-01	2.52735E-01	5.00218E 01	-7.13512E 01

FORCE TRANSMITTED TO BEARING HOUSING

BRG.NO.	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	X-AMPLITUDE	X-PHASE ANG	Y-AMPLITUDE	Y-PHASE ANG
3	8.43212E 00	6.70137E 00	-4.72232E 01	4.34161E 01	7.54912E 00	2.75549E 00	7.68243E 00	-1.02553E 01
16	1.20818E 01	1.14086E 01	-4.16015E 01	4.06316E 01	1.17898E 01	6.54553E-01	1.17101E 01	-2.60526E 00
22	7.86470E-01	8.81354E-02	6.69630E 01	-1.42541E 01	3.18273E-01	1.50982E 02	7.24572E-01	-1.01526E 02
27	3.14685E-01	1.20932E-01	5.17845E 01	-3.71879E 01	2.16621E-01	-6.32039E 01	2.58315E-01	6.96468E 01

ENERGY INPUT= 8.1156011E-04   ENERGY DISSIPATED= 8.1154910E-04

ROTOR SPEED= 3.0000000E 03RPM

	AMPLITUDE				BENDING MOMENT			
STATION	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE
1	2.25094E-01	1.20658E-02	-6.53585E 01	7.06689E 01	0.	0.	0.	0.
					0.	0.	0.	0.
2	2.22603E-01	4.11973E-02	-6.41922E 01	7.58662E 01	1.64000E 01	8.79092E-01	-6.53585E 01	7.06689E 01
					1.64000E 01	8.79092E-01	-6.53585E 01	7.06689E 01
3	2.22490E-01	7.17597E-02	-6.10473E 01	8.25515E 01	3.92504E 01	2.76454E 00	-6.52882E 01	7.13851E 01
					3.92504E 01	2.76454E 00	-6.52882E 01	7.13851E 01
4	2.27303E-01	1.04804E-01	-5.37405E 01	-8.66350E 01	9.93909E 02	1.54111E 02	-2.64767E 00	-2.63695E 01
					9.93909E 02	1.54111E 02	-2.64767E 00	-2.63695E 01
5	2.37895E-01	1.26328E-01	-4.28308E 01	-7.26154E 01	2.20910E 03	3.50130E 02	-2.60601E 00	-2.64721E 01
					2.20910E 03	3.50130E 02	-2.60601E 00	-2.64721E 01
6	2.41836E-01	1.29356E-01	-3.95626E 01	-6.84686E 01	2.60896E 03	4.04079E 02	-2.52290E 00	-2.63567E 01
					2.60896E 03	4.04079E 02	-2.52290E 00	-2.63567E 01
7	2.53622E-01	1.31868E-01	-3.23659E 01	-5.91499E 01	3.54701E 03	4.44692E 02	-1.94096E 00	-2.54486E 01
					3.54701E 03	4.44692E 02	-1.94096E 00	-2.54486E 01
8	2.76831E-01	1.26509E-01	-2.46416E 01	-4.82498E 01	4.62690E 03	1.42101E 02	-2.60662E-01	-2.28475E 01
					4.62690E 03	1.42101E 02	-2.60662E-01	-2.28475E 01
9	2.92022E-01	1.18511E-01	-2.18192E 01	-4.32537E 01	5.07226E 03	3.07036E 02	1.17506E 00	-2.07213E 01
					5.07226E 03	3.07036E 02	1.17506E 00	-2.07213E 01
10	2.93932E-01	1.06700E-01	-2.19377E 01	-4.07470E 01	5.28587E 03	1.03690E 03	2.94853E 00	-1.80797E 01
					5.28587E 03	1.03690E 03	2.94853E 00	-1.80797E 01
11	2.77024E-01	9.64631E-02	-2.54127E 01	-4.16810E 01	4.14490E 03	6.61609E 02	1.73824E 00	-1.85062E 01
					4.14490E 03	6.61609E 02	1.73824E 00	-1.85062E 01
12	2.57255E-01	8.82570E-02	-2.99601E 01	-4.40661E 01	3.27979E 03	4.40411E 02	7.63191E-01	-1.88225E 01
					3.27979E 03	4.40411E 02	7.63191E-01	-1.88225E 01
13	2.27934E-01	7.36702E-02	-3.88397E 01	-4.92543E 01	2.02479E 03	2.11903E 02	-7.36373E-01	-1.91757E 01
					2.02479E 03	2.11903E 02	-7.36373E-01	-1.91757E 01
14	2.07461E-01	5.81651E-02	-4.76062E 01	-5.43440E 01	1.03353E 03	9.13915E 01	-2.34052E 00	-1.92361E 01
					1.03353E 03	9.13915E 01	-2.34052E 00	-1.92361E 01
15	1.99138E-01	4.92886E-02	-5.21555E 01	-5.68621E 01	5.94287E 02	4.40442E 01	-4.32453E 00	-1.92133E 01
					5.94287E 02	4.40442E 01	-4.32453E 00	-1.92133E 01
16	1.88031E-01	3.29341E-02	-5.96711E 01	-6.06176E 01	8.43539E 01	2.02456E 01	3.55589E 01	-2.81718E 01
					8.43539E 01	2.02456E 01	3.55589E 01	-2.81718E 01
17	1.72673E-01	1.07801E-02	-7.65912E 01	-6.48247E 01	6.77890E 01	2.19540E 01	4.39927E 01	-3.28206E 01
					6.77890E 01	2.19540E 01	4.39927E 01	-3.28206E 01
18	1.62530E-01	5.32591E-02	8.37530E 01	-5.99091E 01	4.50871E 01	1.75076E 01	5.24227E 01	-3.87067E 01
					4.50871E 01	1.75076E 01	5.24227E 01	-3.87067E 01
19	1.75160E-02	7.89171E-03	-7.48639E 01	7.89018E 01	2.74958E 01	9.75409E 00	7.08966E 01	-4.92257E 01
					2.74958E 01	9.75409E 00	7.08966E 01	-4.92257E 01
20	1.21598E-02	1.35788E-03	-6.65425E 01	5.68024E 01	3.45910E 01	1.26188E 01	6.86329E 01	-4.80305E 01
					3.45910E 01	1.26188E 01	6.86329E 01	-4.80305E 01
21	1.03870E-02	1.39655E-03	-6.92543E 01	4.47935E 01	4.17231E 01	1.54793E 01	6.71285E 01	-4.72273E 01
					4.17231E 01	1.54793E 01	6.71285E 01	-4.72273E 01
22	8.29596E-03	2.19651E-03	-7.34026E 01	3.77045E 01	6.91418E 01	2.70865E 01	6.45492E 01	-4.63304E 01

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SAMPLE CALCULATION NO.2
ROTOR IN FLEXIBLE PEDESTALS AND GYROSCOPIC MOMENT
  27    4    3    0    1    1    1    1    0    1
 3.1300000E+07  1.0000000E+00
  7            1.000000E-03
 7.600000E+01  3.750000E+00  5.250000E+01  1.282000E+04  7.110000E+03
 2.140000E+01  4.090000E+00  4.880000E+01  0.000000E+00  0.000000E+00
 3.000000E+01  5.310000E+00  4.880000E+01  0.000000E+00  0.000000E+00
 1.250000E+02  6.620000E+00  1.440000E+02  0.000000E+00  0.000000E+00
 1.240000E+02  2.250000E+00  3.020000E+02  0.000000E+00  0.000000E+00
 3.820000E+02  5.910000E+00  7.788000E+02  0.000000E+00  0.000000E+00
 8.760000E+02  9.780000E+00  2.580000E+03  0.000000E+00  0.000000E+00
 7.770000E+02  7.190000E+00  8.380000E+02  0.000000E+00  0.000000E+00
 5.700000E+02  8.570000E+00  3.950000E+02  0.000000E+00  0.000000E+00
 4.530000E+02  7.190000E+00  3.850000E+02  0.000000E+00  0.000000E+00
 2.350000E+02  5.000000E+00  3.210000E+02  0.000000E+00  0.000000E+00
 3.000000E+02  6.620000E+00  6.130000E+02  0.000000E+00  0.000000E+00
 3.130000E+02  4.870000E+00  1.930000E+02  0.000000E+00  0.000000E+00
 8.200000E+01  2.130000E+00  6.800000E+01  0.000000E+00  0.000000E+00
 3.200000E+01  3.190000E+00  4.180000E+01  0.000000E+00  0.000000E+00
 3.672000E+01  7.420000E+00  2.000000E+01  0.000000E+00  0.000000E+00
 2.821000E+01  8.650000E+00  5.476000E+00  1.820000E+02  9.700000E+01
 9.043000E+01  1.237500E+01  3.120000E-01  1.050000E+03  5.450000E+02
 4.297000E+00  1.237500E+00  3.120000E-01  0.000000E+00  0.000000E+00
 4.297000E+00  1.237500E+00  3.120000E-01  0.000000E+00  0.000000E+00
 7.852000E+01  4.750000E+00  2.000000E+01  1.650000E+03  8.800000E+02
 3.423000E+01  5.320000E+00  1.092000E+02  0.000000E+00  0.000000E+00
 9.820000E+01  6.190000E+00  8.540000E+02  0.000000E+00  0.000000E+00
 2.431000E+02  6.510000E+00  1.602000E+03  0.000000E+00  0.000000E+00
 3.300000E+02  6.210000E+00  8.360000E+02  0.000000E+00  0.000000E+00
 2.843000E+02  4.330000E+00  1.554000E+02  0.000000E+00  0.000000E+00
 6.370000E+01  0.000000F+00  0.000000E+00  0.000000E+00  0.000000E+00
   3    16    22    27
   1   1.0000000E+01   0.0000000E+00
  10   1.0000000E+01   1.0000000E+01
  18  -1.0000000E+01   0.0000000E+00
 5.2000E+01  8.2000E+05  2.2000E+01  5.2000E+01  5.3000E+05  2.2000E+01
 5.2000E+01  8.2000E+05  2.2000E+01  5.2000E+01  5.3000E+05  2.2000E+01
 4.4000E+01  1.3000E+06  3.4000E+01  4.4000E+01  9.0000E+05  3.4000E+01
 4.4000E+01  1.3000E+06  3.4000E+01  4.4000E+01  9.0000E+05  3.4000E+01
 9.5000E+01  1.7000E+06  1.8000E+01  9.5000E+01  1.3000E+06  1.8000E+01
 9.5000E+01  1.7000E+06  1.8000E+01  9.5000E+01  1.3000E+06  1.8000E+01
 8.5000E+01  1.9000E+06  2.7000E+01  8.5000E+01  1.5000E+06  2.7000E+01
 8.5000E+01  1.9000E+06  2.7000E+01  8.5000E+01  1.5000E+06  2.7000E+01
 1.000000E+03  5.100000E+03  2.000000E+03
 1.542000E+06  0.000000E+00  0.000000E+00
 3.362000E+06  0.000000E+00  0.000000E+00
-8.010000E+05  0.000000E+00  0.000000E+00
 1.302000E+06  0.000000E+00  0.000000E+00
 1.542000E+05  0.000000E+00  0.000000E+00
 1.040000E+06  0.000000E+00  0.000000E+00
 9.720000E+05  0.000000E+00  0.000000E+00
 1.302000E+06  0.000000E+00  0.000000E+00
 1.542000E+06  0.000000E+00  0.000000E+00
 3.362000E+06  0.000000F+00  0.000000E+00
-8.010000E+05  0.000000E+00  0.000000E+00
 1.302000E+06  0.000000E+00  0.000000E+00
 1.542000E+05  0.000000E+00  0.000000E+00
 1.040000E+06  0.000000E+00  0.000000E+00
 9.720000E+05  0.000000E+00  0.000000E+00
 1.302000E+06  0.000000E+00  0.000000E+00
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1.463000E+06	0.000000E+00	0.000000E+00
3.296000E+06	0.000000E+00	0.000000E+00
-1.045000E+06	0.000000E+00	0.000000E+00
1.781000E+06	0.000000E+00	0.000000E+00
-1.197000E+05	0.000000E+00	0.000000E+00
1.754000E+06	0.000000E+00	0.000000E+00
1.175000E+06	0.000000E+00	0.000000E+00
1.781000E+06	0.000000E+00	0.000000E+00
1.463000E+06	0.000000E+00	0.000000E+00
3.296000E+06	0.000000E+00	0.000000E+00
-1.045000E+06	0.000000E+00	0.000000E+00
1.781000E+06	0.000000E+00	0.000000E+00
-1.197000E+05	0.000000E+00	0.000000E+00
1.754000E+06	0.000000E+00	0.000000E+00
1.175000E+06	0.000000E+00	0.000000E+00
1.781000E+06	0.000000E+00	0.000000E+00
1.070000E+06	0.000000E+00	0.000000E+00
2.240000E+06	0.000000E+00	0.000000E+00
-5.700000E+05	0.000000E+00	0.000000E+00
9.140000E+05	0.000000E+00	0.000000E+00
1.070000E+05	0.000000E+00	0.000000E+00
6.800000E+05	0.000000E+00	0.000000E+00
6.320000E+05	0.000000E+00	0.000000E+00
9.140000E+05	0.000000E+00	0.000000E+00
1.070000E+06	0.000000E+00	0.000000E+00
2.240000E+06	0.000000E+00	0.000000E+00
-5.700000E+05	0.000000E+00	0.000000E+00
9.140000E+05	0.000000E+00	0.000000E+00
1.070000E+05	0.000000E+00	0.000000E+00
6.800000E+05	0.000000E+00	0.000000E+00
6.320000E+05	0.000000E+00	0.000000E+00
9.140000E+05	0.000000F+00	0.000000E+00
9.800000E+05	0.000000F+00	0.000000E+00
2.160000E+06	0.000000E+00	0.000000E+00
-7.000000E+05	0.000000E+00	0.000000E+00
1.330000E+06	0.000000E+00	0.000000E+00
-8.700000E+04	0.000000E+00	0.000000E+00
1.200000E+06	0.000000E+00	0.000000E+00
7.600000E+05	0.000000E+00	0.000000F+00
1.330000E+06	0.000000E+00	0.000000E+00
9.800000E+05	0.000000E+00	0.000000E+00
2.160000E+06	0.000000E+00	0.000000E+00
-7.000000E+05	0.000000E+00	0.000000E+00
1.330000E+06	0.000000E+00	0.000000E+00
-8.700000E+04	0.000000E+00	0.000000E+00
1.200000E+06	0.000000E+00	0.000000E+00
7.600000E+05	0.000000F+00	0.000000E+00
1.330000E+06	0.000000E+00	0.000000E+00

UNBALANCE RESPONSE OF ROTOR WITH NON-UNIFORM BEARING SUPPORTS PN0011
MECHANICAL TECHNOLOGY, INC.

SAMPLE CALCULATION NO.2
ROTOR IN FLEXIBLE PEDESTALS AND GYROSCOPIC MOMENT

STATIONS	NO.BRGS.	NO.UNBAL.	NO.COUPL.	PED.FLEX.	BRG.MOMENT	GYRO.MOM.	NO.CASES	DIAGNOSTIC	INPUT
27	4	3	0	1	1	1	1	0	1

YOUNGS MODULUS	SCALE FACTOR
3.1300000E 07	1.0000000E 00

ITERAT.	ITERAT.CONVERG.LIMIT
7	10.000000E-04

ROTOR DATA

STATION NO.	MASS	LENGTH	CROSS SECT.INERTIA	POLAR MOM.INERTIA	TRANSV.MOM.INERTIA
1	7.6000000E 01	3.7500000E 00	5.2500000E 01	1.2820000E 04	7.1100000E 03
2	2.1400000E 01	4.0900000E 00	4.8799999E 01	0.	0.
3	3.0000000E 01	5.3099999E 00	4.8799999E 01	0.	0.
4	1.2500000E 02	6.6199999E 00	1.4400000E 02	0.	0.
5	1.2400000E 02	2.2500000E 00	3.0200000E 02	0.	0.
6	3.8200000E 02	5.9100000E 00	7.7879999E 02	0.	0.
7	8.7600000E 02	9.7800000E 00	2.5800000E 03	0.	0.
8	7.7700000E 02	7.1900000E 00	8.3800000E 02	0.	0.
9	5.7000000E 02	8.5699999E 00	3.9500000E 02	0.	0.
10	4.5300000E 02	7.1900000E 00	3.8500000E 02	0.	0.
11	2.3500000E 02	5.0000000E 00	3.2100000E 02	0.	0.
12	3.0000000E 02	6.6199999E 00	6.1300000E 02	0.	0.
13	3.1300000E 02	4.8699999E 00	1.9300000E 02	0.	0.
14	8.1999999E 01	2.1300000E 00	6.8000000E 01	0.	0.
15	3.2000000E 01	3.1900000E 00	4.1799999E 01	0.	0.
16	3.6720000E 01	7.4200000E 00	2.0000000E 01	0.	0.
17	2.8210000E 01	8.6500000E 00	5.4760000E 00	1.6200000E 02	9.6999999E 01
18	9.0429999E 01	1.2375000E 01	3.1200000E-01	1.0500000E 03	5.4500000E 02
19	4.2970000E 00	1.2375000E 00	3.1200000E-01	0.	0.
20	4.2970000E 00	1.2375000E 00	3.1200000E-01	0.	0.
21	7.8519999E 01	4.7500000E 00	2.0000000E 01	1.6500000E 03	8.8000000E 02
22	3.4229999E 01	5.3200000E 00	1.0920000E 02	0.	0.
23	9.8199999E 01	6.1900000E 00	8.5400000E 02	0.	0.
24	2.4310000E 02	6.5100000E 00	1.6020000E 03	0.	0.
25	3.3000000E 02	6.2100000E 00	8.3599999E 02	0.	0.
26	2.8430000E 02	4.3300000E 00	1.5540000E 02	0.	0.
27	6.3699999E 01	0.	0.	0.	0.

BEARING STATIONS
3 16 22 27

UNBALANCE ST.	X-UNBALANCE	Y-UNBALANCE
1	1.0000000E 01	0.
10	1.0000000E 01	1.0000000E 01
18	-1.0000000E 01	0.

PEDESTAL DATA, TRANSLATORY MOTION

BRG.STATION	MASS,X-DIR.	KX	CX	MASS,Y-DIR.	KY	CY
3	5.2000E 01	8.2000E 05	2.2000E 01	5.2000E 01	5.3000E 05	2.2000E 01
16	5.2000E 01	8.2000E 05	2.2000E 01	5.2000E 01	5.3000E 05	2.2000E 01
22	4.4000E 01	1.3000E 06	3.4000E 01	4.4000E 01	9.0000E 05	3.4000E 01
27	4.4000E 01	1.3000E 06	3.4000E 01	4.4000E 01	9.0000E 05	3.4000E 01

PEDESTAL DATA, ROTATIONAL MOTION

BRG.STATION	INERTIA,X	MX	DX	INERTIA,Y	MY	DY

3	9.5000E 01	1.7000E 06	1.8000E 01	9.5000E 01	1.3000E 06	1.8000E 01
16	9.5000E 01	1.7000E 06	1.8000E 01	9.5000E 01	1.3000E 06	1.8000E 01
22	8.5000E 01	1.9000E 06	2.7000E 01	8.5000E 01	1.5000E 06	2.7000E 01
27	8.5000E 01	1.9000E 06	2.7000E 01	8.5000E 01	1.5000E 06	2.7000E 01

INITIAL SPEED	FINAL SPEED	SPEED INCR.
1.000000E 03	5.100000E 03	2.000000E 03

BEARING AT STATION NO. 3

KXX	CXX	KXY	CXY	KYY	CYY	KYX	CYX
1.542000E 06	3.362000E 06	-8.010000E 05	1.302000E 06	1.542000E 05	1.040000E 06	9.720000E 05	1.302000E 06
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

BEARING AT STATION NO. 16

KXX	CXX	KXY	CXY	KYY	CYY	KYX	CYX
1.542000E 06	3.362000E 06	-8.010000E 05	1.302000E 06	1.542000E 05	1.040000E 06	9.720000E 05	1.302000E 06
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

BEARING AT STATION NO. 22

KXX	CXX	KXY	CXY	KYY	CYY	KYX	CYX
1.463000E 06	3.296000E 06	-1.045000E 06	1.781000E 06	-1.197000E 05	1.754000E 06	1.175000E 06	1.781000E 06
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

BEARING AT STATION NO. 27

KXX	CXX	KXY	CXY	KYY	CYY	KYX	CYX
1.463000E 06	3.296000E 06	-1.045000E 06	1.781000E 06	-1.197000E 05	1.754000E 06	1.175000E 06	1.781000E 06
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

BEARING AT STATION NO. 3

MXX	DXX	MXY	DXY	MYY	DYY	MYX	DYX
1.070000E 06	2.240000E 06	-5.700000E 05	9.140000E 05	1.070000E 05	6.800000E 05	6.320000E 05	9.140000E 05
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

BEARING AT STATION NO. 16

MXX	DXX	MXY	DXY	MYY	DYY	MYX	DYX
1.070000E 06	2.240000E 06	-5.700000E 05	9.140000E 05	1.070000E 05	6.800000E 05	6.320000E 05	9.140000E 05
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

BEARING AT STATION NO. 22

MXX	DXX	MXY	DXY	MYY	DYY	MYX	DYX
9.800000E 05	2.160000E 06	-7.000000E 05	1.330000E 06	-8.700000E 04	1.200000E 06	7.600000E 05	1.330000E 06
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

BEARING AT STATION NO. 27

MXX	DXX	MXY	DXY	MYY	DYY	MYX	DYX
9.800000E 05	2.160000E 06	-7.000000E 05	1.330000E 06	-8.700000E 04	1.200000E 06	7.600000E 05	1.330000E 06
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

ROTOR SPEED= 1.0000000E 03RPM
WITHOUT GYROSCOPIC MOMENT

	AMPLITUDE				BENDING MOMENT			
STATION	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE
1	1.33225E-01	3.16944E-02	-6.60054E 01	5.63500E 01	0.	0.	0.	0.

					0.	0.	0.	0.
2	1.25896E-01	2.85316E-02	-6.55021E 01	5.77223E 01	6.76430E 01	6.68211E 01	-6.12254E 01	6.11300E 01
					6.76430E 01	6.68211E 01	-6.12254E 01	6.11300E 01
3	1.18738E-01	2.58807E-02	-6.48160E 01	5.94446E 01	1.41730E 02	1.39770E 02	-6.12843E 01	6.11784E 01
					1.41139E 02	1.38330E 02	-5.88326E 01	5.88597E 01
4	1.11556E-01	2.45263E-02	-6.37680E 01	6.19861E 01	8.28034E 01	6.43809E 01	2.97671E 00	-3.72063E 01
					8.28034E 01	6.43809E 01	2.97671E 00	-3.72063E 01
5	1.04061E-01	2.44543E-02	-6.25644E 01	6.53146E 01	1.21149E 02	7.84904E 01	-3.48981E 01	-7.61854E 01
					1.21149E 02	7.84904E 01	-3.48981E 01	-7.61854E 01
6	1.01562E-01	2.45187E-02	-6.22053E 01	6.64794E 01	1.55203E 02	1.04926E 02	-4.01891E 01	-8.14789E 01
					1.55203E 02	1.04926E 02	-4.01891E 01	-8.14789E 01
7	9.51002E-02	2.47787E-02	-6.13255E 01	6.97453E 01	2.49021E 02	1.83591E 02	-4.65523E 01	-8.70929E 01
					2.49021E 02	1.83591E 02	-4.65523E 01	-8.70929E 01
8	8.49679E-02	2.55549E-02	-6.00806E 01	7.60979E 01	3.90840E 02	3.17171E 02	-5.06405E 01	-8.85924E 01
					3.90840E 02	3.17171E 02	-5.06405E 01	-8.85924E 01
9	7.79301E-02	2.61943E-02	-5.95314E 01	8.17384E 01	4.83485E 02	4.13001E 02	-5.29896E 01	-8.80274E 01
					4.83485E 02	4.13001E 02	-5.29896E 01	-8.80274E 01
10	6.87416E-02	2.57240E-02	-5.97312E 01	8.99130E 01	5.83796E 02	5.23995E 02	-5.57007E 01	-8.63074E 01
					5.83796E 02	5.23995E 02	-5.57007E 01	-8.63074E 01
11	5.98308E-02	2.33887E-02	-6.07596E 01	-8.11220E 01	4.83570E 02	4.36315E 02	-6.02052E 01	-8.52849E 01
					4.83570E 02	4.36315E 02	-6.02052E 01	-8.52849E 01
12	5.33322E-02	2.07218E-02	-6.17752E 01	-7.29562E 01	4.13489E 02	3.75775E 02	-6.48827E 01	-8.40188E 01
					4.13489E 02	3.75775E 02	-6.48827E 01	-8.40188E 01
13	4.56908E-02	1.65982E-02	-6.28493E 01	-5.86551E 01	3.21606E 02	2.97202E 02	-7.56813E 01	-8.00412E 01
					3.21606E 02	2.97202E 02	-7.56813E 01	-8.00412E 01
14	4.16863E-02	1.36546E-02	-6.23888E 01	-4.50630E 01	2.58069E 02	2.42211E 02	8.73962E 01	-7.12170E 01
					2.58069E 02	2.42211E 02	8.73962E 01	-7.12170E 01
15	4.05533E-02	1.25348E-02	-6.13304E 01	-3.80842E 01	2.33229E 02	2.19385E 02	7.51608E 01	-6.38427E 01
					2.33229E 02	2.19385E 02	7.51608E 01	-6.38427E 01
16	4.00537E-02	1.15729E-02	-5.78799E 01	-2.59493E 01	2.01817E 02	1.87290E 02	5.37575E 01	-5.16522E 01
					2.03681E 02	1.89223E 02	4.94473E 01	-4.71926E 01
17	5.26825E-02	2.14223E-02	-4.47954E 01	4.48914E 00	9.61907E 01	8.57514E 01	5.01700E 01	-4.67309E 01
					9.61907E 01	8.57514E 01	5.01700E 01	-4.67309E 01
18	9.55763E-02	6.06187E-02	-3.93508E 01	2.39909E 01	3.48735E 01	2.92789E 01	-4.41375E 01	3.94100E 01
					3.48735E 01	2.92789E 01	-4.41375E 01	3.94100E 01
19	2.65367E-02	8.16999E-03	-4.39028E 01	2.23150E 01	1.58334E 01	1.33900E 01	-3.98708E 01	3.18475E 01
					1.58334E 01	1.33900E 01	-3.98708E 01	3.18475E 01
20	2.04160E-02	3.61695E-03	-4.65381E 01	1.92895E 01	2.08987E 01	1.76585E 01	-4.06097E 01	3.31326E 01
					2.08987E 01	1.76585E 01	-4.06097E 01	3.31326E 01
21	1.75015E-02	1.76649E-03	-4.84888E 01	1.71386E 01	2.59602E 01	2.19274E 01	-4.10528E 01	3.39076E 01
					2.59602E 01	2.19274E 01	-4.10528E 01	3.39076E 01
22	1.36893E-02	8.23797E-04	-4.96929E 01	1.57895E 01	4.56134E 01	3.83234E 01	-4.17221E 01	3.51048E 01
					4.53666E 01	3.79237E 01	-4.25678E 01	3.58612E 01
23	1.02957E-02	4.93619E-04	-5.00512E 01	1.53511E 01	3.66670E 01	3.09179E 01	-4.27458E 01	3.61829E 01
					3.66670E 01	3.09179E 01	-4.27458E 01	3.61829E 01
24	6.53956E-03	2.67578E-04	-5.01949E 01	1.50804E 01	2.67084E 01	2.27659E 01	-4.29379E 01	3.65580E 01
					2.67084E 01	2.27659E 01	-4.29379E 01	3.65580E 01
25	2.61837E-03	5.53565E-05	-5.05375E 01	1.42633E 01	1.65120E 01	1.41934E 01	-4.25837E 01	3.63430E 01
					1.65120E 01	1.41934E 01	-4.25837E 01	3.63430E 01
26	1.10936E-03	1.35365E-04	-4.93747E 01	1.83733E 01	6.91542E 00	6.00630E 00	-4.06218E 01	3.45657E 01
					6.91542E 00	6.00630E 00	-4.06218E 01	3.45657E 01
27	3.69120E-03	2.55958E-04	-4.99562E 01	1.62719E 01	3.09563E-01	1.79296E-01	2.93963E 01	-2.59627E 01
					2.67749E-02	3.56188E-03	-6.22862E 01	-8.46474E 00

FORCE TRANSMITTED TO BEARING HOUSING

BRG.NO.	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	X-AMPLITUDE	X-PHASE ANG	Y-AMPLITUDE	Y-PHASE ANG
3	3.59537E 01	3.16414E 01	-6.12504E 01	7.45352E 01	3.26897E 01	1.64737E 01	3.50033E 01	1.03069E 01
16	1.43427E 01	1.14576E 01	-9.31719E 00	-5.48981E 01	1.42746E 01	1.17635E 02	1.15425E 01	1.13496E 02
22	5.78382E 00	4.76906E 00	-4.22793E 01	3.59292E 01	5.34849E 00	1.79069E 02	5.25266E 00	1.68132E 02
27	1.55659E 00	1.31863E 00	-4.42143E 01	3.77054E 01	1.44577E 00	-1.79021E 00	1.43927E 00	-1.12489E 01

FORCE TRANSMITTED TO FOUNDATION

BRG.NO.	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	X-AMPLITUDE	X-PHASE ANG	Y-AMPLITUDE	Y-PHASE ANG
3	3.60461E 01	3.17056E 01	-6.14366E 01	7.47193E 01	3.27486E 01	1.64734E 01	3.51011E 01	1.03062E 01
16	1.43690E 01	1.14893E 01	-9.35915E 00	-5.48576E 01	1.43003E 01	1.17635E 02	1.15747E 01	1.13495E 02
22	5.79051E 00	4.77475E 00	-4.23434E 01	3.59919E 01	5.35360E 00	1.79069E 02	5.25997E 00	1.68132E 02
27	1.55841E 00	1.32019E 00	-4.42889E 01	3.77787E 01	1.44717E 00	-1.79036E 00	1.44127E 00	-1.12493E 01

PEDESTAL MOTION

BRG.NO.	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	X-AMPLITUDE	X-PHASE ANG	Y-AMPLITUDE	Y-PHASE ANG
3	6.64499E-02	3.95669E-02	-8.41662E 01	-8.34243E 01	3.99372E-02	1.63124E 01	6.62280E-02	1.00571E 01
16	2.19405E-02	1.73114E-02	-8.09993E 01	1.61226E 01	1.74394E-02	1.17474E 02	2.18389E-02	1.13246E 02
22	5.94201E-03	3.97596E-03	-7.59399E 01	6.83918E 01	4.11814E-03	1.78912E 02	5.84436E-03	1.67905E 02
27	1.62087E-03	1.08465E-03	-7.79971E 01	7.04267E 01	1.11320E-03	-1.94729E 00	1.60140E-03	-1.14759E 01

ENERGY INPUT= 5.7557979E-03 ENERGY DISSIPATED= 5.6888012E-03

ITERAT.NO.	ERROR
1	1.0000000E 00
2	1.7304529E-06

WITH GYROSCOPIC MOMENT

	AMPLITUDE				BENDING MOMENT			
STATION	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE
1	1.33219E-01	3.16726E-02	-6.60011E 01	5.63507E 01	0.	0.	0.	0.
					2.95442E-01	5.62445E-02	2.12196E 01	-5.58861E 01
2	1.25891E-01	2.65193E-02	-6.54993E 01	5.77229E 01	6.76125E 01	6.65619E 01	-5.90062E 01	5.89961E 01
					6.76125E 01	6.65619E 01	-5.90062E 01	5.89961E 01
3	1.18734E-01	2.58759E-02	-6.48147E 01	5.94451E 01	1.41699E 02	1.39511E 02	-6.02130E 01	6.01478E 01
					1.41111E 02	1.38071E 02	-5.82349E 01	5.83030E 01
4	1.11553E-01	2.45256E-02	-6.37680E 01	6.19866E 01	8.25551E 01	6.43095E 01	2.76761E 00	-3.69939E 01
					8.25551E 01	6.43095E 01	2.76761E 00	-3.69939E 01
5	1.04058E-01	2.44569E-02	-6.25653E 01	6.53150E 01	1.21028E 02	7.85333E 01	-3.50178E 01	-7.61436E 01
					1.21028E 02	7.85333E 01	-3.50178E 01	-7.61436E 01
6	1.01559E-01	2.45215E-02	-6.22065E 01	6.64799E 01	1.55110E 02	1.04991E 02	-4.02850E 01	-8.14421E 01
					1.55110E 02	1.04991E 02	-4.02850E 01	-8.14421E 01
7	9.50968E-02	2.47821E-02	-6.13274E 01	6.97459E 01	2.48958E 02	1.83671E 02	-4.66152E 01	-8.70612E 01
					2.48958E 02	1.83671E 02	-4.66152E 01	-8.70612E 01
8	8.49842E-02	2.55584E-02	-6.00839E 01	7.60989E 01	3.90792E 02	3.17237E 02	-5.08850E 01	-8.85625E 01
					3.90792E 02	3.17237E 02	-5.08850E 01	-8.85625E 01
9	7.79261E-02	2.61976E-02	-5.95359E 01	8.17400E 01	4.83442E 02	4.13049E 02	-5.30287E 01	-8.79974E 01
					4.83442E 02	4.13049E 02	-5.30287E 01	-8.79974E 01
10	6.87375E-02	2.57264E-02	-5.97374E 01	8.99158E 01	5.83757E 02	5.24019E 02	-5.57369E 01	-8.62761E 01
					5.83757E 02	5.24019E 02	-5.57369E 01	-8.62761E 01
11	5.98270E-02	2.33899E-02	-6.07675E 01	-8.11177E 01	4.83536E 02	4.36320E 02	-6.02409E 01	-8.52529E 01
					4.83536E 02	4.36320E 02	-6.02409E 01	-8.52529E 01
12	5.33287E-02	2.07219E-02	-6.17842E 01	-7.29509E 01	4.13457E 02	3.75766E 02	-6.49175E 01	-8.39864E 01
					4.13457E 02	3.75766E 02	-6.49175E 01	-8.39864E 01
13	4.56881E-02	1.65967E-02	-6.29591E 01	-5.86486E 01	3.21573E 02	2.97173E 02	-7.57109E 01	-8.00121E 01
					3.21573E 02	2.97173E 02	-7.57109E 01	-8.00121E 01
14	4.16843E-02	1.36519E-02	-6.23980E 01	-4.50562E 01	2.58031E 02	2.42168E 02	8.73868E 01	-7.12069E 01
					2.58031E 02	2.42168E 02	8.73868E 01	-7.12069E 01
15	4.05517E-02	1.25317E-02	-6.13390E 01	-3.80776E 01	2.33185E 02	2.19338E 02	7.51697E 01	-6.38507E 01
					2.33185E 02	2.19338E 02	7.51697E 01	-6.38507E 01
16	4.00525E-02	1.15692E-02	-5.78876E 01	-2.59431E 01	2.01758E 02	1.87241E 02	5.37799E 01	-5.18742E 01
					2.03621E 02	1.89175E 02	4.94668E 01	-4.72117E 01
17	5.26796E-02	2.14153E-02	-4.48021E 01	4.49340E 00	9.61297E 01	8.56927E 01	5.01977E 01	-4.67579E 01
					9.61408E 01	8.57010E 01	5.01928E 01	-4.67530E 01
18	9.55551E-02	6.05908E-02	-3.93596E 01	2.39953E 01	3.49358E 01	2.93492E 01	-4.40858E 01	3.93761E 01
					3.48601E 01	2.92647E 01	-4.41453E 01	3.94124E 01
19	2.65274E-02	8.15534E-03	-4.39144E 01	2.23249E 01	1.58290E 01	1.33867E 01	-3.98659E 01	3.18346E 01
					1.58290E 01	1.33867E 01	-3.98659E 01	3.18346E 01
20	2.04086F-02	3.60518E-03	-4.65499E 01	1.92985E 01	2.08924E 01	1.76535E 01	-4.06075E 01	3.31228E 01
					2.08924E 01	1.76535E 01	-4.06075E 01	3.31228E 01

21	1.74952E-02	1.75687E-13	-4.84999E 01	1.71455E 01	2.59611E 01	2.19206E 01	-4.10522E 01	3.38996E 01
					2.59477E 01	2.18031E 01	-4.14884E 01	3.43997E 01
22	1.36848E-02	8.18990E-14	-4.97009E 01	1.57928E 01	4.55861E 01	3.81916E 01	-4.19551E 01	3.53679E 01
					4.53403E 01	3.77918E 01	-4.27828E 01	3.61064E 01
23	1.02923E-02	4.91101E-14	-5.00592E 01	1.53513E 01	3.66466E 01	3.08126E 01	-4.29695E 01	3.64386E 01
					3.66466E 01	3.08126E 01	-4.29695E 01	3.64386E 01
24	6.53644E-03	2.67051E-14	-5.02062E 01	1.50737E 01	2.66966E 01	2.26920E 01	-4.31714E 01	3.68300E 01
					2.66966E 01	2.26920E 01	-4.31714E 01	3.68300E 01
25	2.61675E-03	5.51314E-15	-5.05356E 01	1.42302E 01	1.65037E 01	1.41499E 01	-4.28008E 01	3.66044E 01
					1.65037E 01	1.41499E 01	-4.28008E 01	3.66044E 01
26	1.10909E-03	1.32700E-14	-4.94346E 01	1.84410E 01	6.91037E 00	5.99726E 00	-4.09340E 01	3.49016E 01
					6.91037E 00	5.99726E 00	-4.09340E 01	3.49016E 01
27	3.69177E-03	2.52072E-14	-4.99588E 01	1.62904E 01	3.35195E-01	1.82294E-01	2.98148E 01	-2.64737E 01
					2.76164E-02	7.85696E-03	-8.60542E 01	-3.20458E 01

FORCE TRANSMITTED TO BEARING HOUSING

BRG.NO.	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	X-AMPLITUDE	X-PHASE ANG	Y-AMPLITUDE	Y-PHASE ANG
3	3.59526E 01	3.16379E 01	-6.12385E 01	7.45264E 01	3.26875E 01	1.64796E 01	3.50010E 01	1.03073E 01
15	1.43389E 01	1.14572E 01	-9.33820E 00	-5.48788E 01	1.42705E 01	1.17636E 02	1.15423E 01	1.13492E 02
22	5.78120E 00	4.76236E 00	-4.23377E 01	3.59919E 01	5.34319E 00	1.79101E 02	5.24908E 00	1.68109E 02
27	1.55576E 00	1.31302E 00	-4.43207E 01	3.78501E 01	1.44238E 00	-1.64518E 00	1.43665E 00	-1.13152E 01

FORCE TRANSMITTED TO FOUNDATION

BRG.NO.	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	X-AMPLITUDE	X-PHASE ANG	Y-AMPLITUDE	Y-PHASE ANG
3	3.60450E 01	3.17021E 01	-6.14246E 01	7.47105E 01	3.27465E 01	1.64793E 01	3.50984E 01	1.03066E 01
15	1.43652E 01	1.14889E 01	-9.38029E 00	-5.48382E 01	1.42963E 01	1.17636E 02	1.15745E 01	1.13491E 02
22	5.78769E 00	4.76806E 00	-4.24015E 01	3.60542E 01	5.34833E 00	1.79101E 02	5.25638E 00	1.68109E 02
27	1.55758E 00	1.31457E 00	-4.43936E 01	3.79218E 01	1.44377E 00	-1.64533E 00	1.43864E 00	-1.13155E 01

PEDESTAL MOTION

BRG.NO.	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	X-AMPLITUDE	X-PHASE ANG	Y-AMPLITUDE	Y-PHASE ANG
3	6.64459E-02	3.95636E-02	-8.41611E 01	-8.34269E 01	3.99344E-02	1.63184E 01	6.62234E-02	1.08575E 01
15	2.19402E-02	1.73063E-02	-8.11001E 01	1.61211E 01	1.74344E-02	1.17475E 02	2.18385E-02	1.13242E 02
22	5.93874E-03	3.97076E-03	-7.58917E 01	6.83427E 01	4.11408E-03	1.78944E 02	5.84037E-03	1.67882E 02
27	1.61870E-03	1.08091E-03	-7.77803E 01	7.02290E 01	1.11059E-03	-1.80225E 00	1.59848E-03	-1.15422E 01

ENERGY INPUT= 5.7550767E-03   ENERGY DISSIPATED= 5.6882025E-03

ROTOR SPEED= 3.0000000E 03RPM
WITHOUT GYROSCOPIC MOMENT

	AMPLITUDE				BENDING MOMENT			
STATION	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE	MAJOR AXIS	MINOR AXIS	ANGLE X-MAJOR	PHASE ANGLE
1	1.42189E 00	8.28902E-01	7.72963E 01	1.97919E 01	0.	0.	0.	0.
					0.	0.	0.	0.
2	1.43686E 00	8.97328E-01	6.57896E 01	2.69452E 01	6.36294E 02	5.93090E 02	-6.50512E 01	5.74445E 01
					6.36294E 02	5.93090E 02	-6.50512E 01	5.74445E 01
3	1.49822E 00	9.42619E-01	5.28259E 01	3.58316E 01	1.34098E 03	1.23914E 03	-6.67085E 01	5.79606E 01
					1.34028E 03	1.23617E 03	-8.03346E 01	7.03224E 01
4	1.62357E 00	9.53307E-01	3.99005E 01	4.48781E 01	5.54723E 03	1.86739E 03	1.10309E 01	5.03506E 01
					5.54723E 03	1.86739E 03	1.10309E 01	5.03506E 01
5	1.75572E 00	9.04685E-01	3.03949E 01	5.09714E 01	1.17965E 04	3.59872E 03	1.64658E 01	5.54996E 01
					1.17965E 04	3.59872E 03	1.64658E 01	5.54996E 01
6	1.78793E 00	8.77968E-01	2.30907E 01	5.22591E 01	1.38221E 04	4.14637E 03	1.71160E 01	5.62266E 01
					1.38221E 04	4.14637E 03	1.71160E 01	5.62266E 01
7	1.86119E 00	7.97000E-01	2.31070E 01	5.46367E 01	1.81386E 04	5.08696E 03	1.77229E 01	5.75548E 01
					1.81386E 04	5.08696E 03	1.77229E 01	5.75548E 01
8	1.97053E 00	6.47305E-01	1.68098E 01	5.65788E 01	2.12390E 04	4.89722E 03	1.76054E 01	5.93243E 01
					2.12390E 04	4.89722E 03	1.76054E 01	5.93243E 01
9	2.03254E 00	5.29588E-01	1.30000E 01	5.70149E 01	2.07219E 04	3.84041E 03	1.78037E 01	6.08133E 01
					2.07219E 04	3.84041E 03	1.78037E 01	6.08133E 01
10	2.03518E 00	3.78020E-01	8.59503E 00	5.66834E 01	1.76221E 04	1.96383E 03	1.91046E 01	6.35993E 01

INPUT FOR PN0011:

UNBALANCE RESPONSE OF A FLEXIBLE ROTOR IN FLEXIBLE, DAMPED BEARINGS

Card 1   Text Col. 2-72

Card 2   Text Col. 2-72

Card 3   (10I5)

_____ 1. NS. Number of rotor mass stations ($\leq$ 80)
_____ 2. NB. Number of bearings ($\leq$ 25)
_____ 3. NU. Number of unbalance stations ($\leq$ 80)
_____ 4. NC. Number of coupling stations ($\leq$ 20)
_____ 5. NPST. 0: Rigid Pedestal   1: Flexible pedestal
_____ 6. NMOM. 0: No bearing resistance to moment 1: Moment resistance in brgs.
_____ 7. NGYR 0: No gyroscopic moment 1: Gyroscopic moment calculation
_____ 8. NCAL 1: 1st type of bearing data input $\geq$2: 2nd type of bearing data input.
_____ 9. 0: no diagnostic   1: diagnostic given
_____10. 0: More input follows   1: last set of input

Card 4 (1P4E15.7)

_____________________ 1. E, Youngs modulus, lbs/in^2
_____________________ 2. Scale factor in simultaneous equation solution

IF NGYR = 1

Card (I5,1PE23.6)

_____ 1. NIT. Number of iterations in gyroscopic mom.Calc.
_____________________ 2. Convergence limit for gyroscopic moment calcul.

ROTOR DATA

If NGYR = 0, use only first 3 columns, FORMAT (1P3E14.6)

If NGYR = 1, use all 5 columns, FORMAT (1P5E14.6)

Give one card for each rotor station, in total NS cards

Rotor Station (don't punch)	Station Mass lbs.	Length of shaft section inch	Cross sectional Moment of Inertia in^4	Polar Mass Moment of Inertia $lbs.in^2$	Transverse Mass Moment of Inertia $lbs.in^2$
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					

• Rotor Stations with Bearing Support

(14I5)

Give NB items

____

Unbalance Data
(I5,1P2E15.7)

Give one card for each rotor station with unbalance, in total NU cards

__________. 1. Rotor station number

__________________________. 2. X-component of unbalance, oz.inch

__________________________. 3. Y-component of unbalance, oz.inch

Rotor Stations with Coupling

(14I5)

Applies only if NC ≠ 0. Give NC items

Pedestal Data for Translatory Motion.

(1P6E12.4)

Applies only when NPST = 1. Give one card for each bearing, in total NB cards

Pedes.Mass x-direction lbs.	Pedes.Stiffn. x-direction lbs/in	Pedes.Damping x-direction lbs.sec/in	Pedes.Mass. y-direction lbs.	Pedes.Stiffn. y-direction lbs./in	Pedes. Damping y-direction lbs.sec/in

Pedestal Data for Tilting

(1P6E12.4)

Applies only when NPST = 1 and NMOM = 1. Give on card for each bearing, in total NB cards.

Mass Mom. of Inert. x-direction $lbs.in^2$	Angular Stiffn. x-direction lbs.in/rad	Angular Damping x-direction lbs.in.sec/rad	Mass Mom. of Inert. y-direction $lbs.in^2$	Angular Stiffn. y-direction lbs.in/rad	Angular Damping y-direction lbs.in.sec/rad

Type 1 Bearing Data, NCAL = 1

Speed Data (1P3E14.6)

______________________ 1. Initial speed, RPM

______________________ 2. Final speed, RPM

______________________ 3. Speed increment, RPM

Bearing Coefficients for Translatory Motion
(1P3E14.6)

Give 8 cards per bearing, in total 8·NB cards. Each card gives one coefficient in the form: $K_{xx} = K_{xx,0} + K_{xx,1}\,\omega + K_{xx,2}\,\omega^2$, $\omega C_{xx} = C_{xx,0} + C_{xx,1}\,\omega + C_{xx,2}\,\omega^2$, etc.

__________	__________	__________	K_{xx}
__________	__________	__________	ωC_{xx}
__________	__________	__________	K_{xy}
__________	__________	__________	ωC_{xy}
__________	__________	__________	K_{yy}
__________	__________	__________	ωC_{yy}
__________	__________	__________	K_{yx}
__________	__________	__________	ωC_{yx}

__________	__________	__________	K_{xx}
__________	__________	__________	ωC_{xx}
__________	__________	__________	K_{xy}
__________	__________	__________	ωC_{xy}
__________	__________	__________	K_{yy}
__________	__________	__________	ωC_{yy}
__________	__________	__________	K_{yx}
__________	__________	__________	ωC_{yx}

<u>Bearing Coefficients for Tilting</u>

(1P3E14.6)

Applies only when NMOM = 1. Give 8 cards per bearing, in total 8·NB cards. Each card gives one coefficient in the form:

$M_{xx} = M_{xx,0} + M_{xx,1}\,\omega + M_{xx,2}\,\omega^2$, $\omega D_{xx} = D_{xx,0} + D_{xx,1}\,\omega + D_{xx,2}\,\omega^2$, etc.

______	______	______	M_{xx}
______	______	______	ωD_{xx}
______	______	______	M_{xy}
______	______	______	ωD_{xy}
______	______	______	M_{yy}
______	______	______	ωD_{yy}
______	______	______	M_{yx}
______	______	______	ωD_{yx}

______	______	______	M_{xx}
______	______	______	ωD_{xx}
______	______	______	M_{xy}
______	______	______	ωD_{xy}
______	______	______	M_{yy}
______	______	______	ωD_{yy}
______	______	______	M_{yx}
______	______	______	ωD_{yx}

Type 2 Bearing Data, NCAL $\geq$ 2.

Repeat the following input as many times as given by NCAL.

Speed Data (1P4E14.6)

___________________________________. Speed, RPM

Bearing Coefficients for Translatory Motion

(1P4E14.6)

Give 2 cards per bearing with 4 coefficients per card, in total 2•NB Cards

___________ ___________ ___________ ___________ $K_{xx}, \omega C_{xx}, K_{xy}, \omega C_{xy}$
___________ ___________ ___________ ___________ $K_{yy}, \omega C_{yy}, K_{yx}, \omega C_{yx}$

___________ ___________ ___________ ___________ $K_{xx}, \omega C_{xx}, K_{xy}, \omega C_{xy}$
___________ ___________ ___________ ___________ $K_{yy}, \omega C_{yy}, K_{yx}, \omega C_{yx}$

Bearing Coefficients for Tilting

(1P4E14.6)

Applies only when NMOM=1. Give 2 cards per bearing, in total 2•NB Cards

___________ ___________ ___________ ___________ $M_{xx}, \omega D_{xx}, M_{xy}, \omega D_{xy}$
___________ ___________ ___________ ___________ $M_{yy}, \omega D_{yy}, M_{yx}, \omega D_{yx}$

___________ ___________ ___________ ___________ $M_{xx}, \omega D_{xx}, M_{xy}, \omega D_{xy}$
___________ ___________ ___________ ___________ $M_{yy}, \omega D_{yy}, M_{yx}, \omega D_{yx}$

VOLUME 3

PART II

EXPERIMENTAL INVESTIGATION

OF

HYDRAULIC SUPPORTS

By

Ronald J. Thoman
Westinghouse Electric Corporation

TABLE OF CONTENTS

ABSTRACT

Laboratory tests proved the noise reduction ability of hydraulic-supports when applied under the bearings of a high-speed rotor. The results were compared with calculated values as well as with tests conducted with rigid-supports. Curves presented show that the force transmitted to the bedplate was reduced by a factor as great as 15 over that for rigid-supports.

INTRODUCTION

An experimental investigation was conducted into the ability of a hydraulically-supported, pivoted-shoe, journal bearing to reduce structureborne noise resulting from rotor unbalance.

High-speed rotating machinery onboard submarines represents a principle noise source. Because of the demand for quiet operation, equipment such as main propulsion turbines and turbine-generator sets are built to stringent manufacturing specifications, isolated with resilient mounts and balanced using the most refined techniques. Still a relentless search for more silent rotating equipment continues.

As part of a "Study of Noise Sources in 800 KW Turbine-Generator Sets", Hagaman[1] analyzed one promising concept which involved flexibly supporting a rotor directly at its bearings, close to the noise source. He concluded that a large reduction in transmitted force appeared attainable with sufficiently soft supports. Since the bearings were in the direct transmission path, the noise originating in the rotor must pass through the bearings to excite the bedplate and surrounding structure. Therefore, bearing load variation (transmitted force) has a direct relation to the structureborne noise.

Lund, Sternlicht, and McHugh[2,3] demonstrated with mathematical analyses and laboratory tests that the oil-film in hydrodynamic bearings had spring-like properties and thus acted as a flexible support with the potential of reducing the noise transmission. However, drastic reduction seemed possible only by using an external support system.

Coil-springs held the bearing in a study conducted by Fistere and Dickson[4] to examine the feasibility of the flexibly-supported bearing for submarine propulsion turbines. Since shipboard operation required correcting for the variation of bearing load, which would cause excessive deflections because of the low spring-value needed for isolating, accent was placed on the development of an automatic positioning system. Data gathered from extensive laboratory testing proved the soft-support scheme practical and an excellent method of reducing structureborne vibrations.

[1] Numbers placed superior to the line refer to the Bibliography.

The present contract advanced the knowledge of the flexible-support concept. The analytical work was explained in other parts of this report. This portion is devoted to the experimental investigation, which had two principle purposes:

1. To determine the feasibility and desirability of hydraulic-supports for isolating a rotor-bearing system.

2. To substantiate the analytical tools developed under this contract.

Compared to mechanical springs, hydraulic-supports are potentially space-savers since accumulators, which can be placed remote from the bearings, provide the flexibility. As an additional advantage, the servo-return system can be integrated into the hydraulic-supports.

The laboratory model was not designed as a shipboard prototype, but emphasis was placed on determining valid and meaningful measurements of the isolation ability of the supports. No servo-return system was included since this was examined in the previous report[4].

With a known unbalance, an unbladed SS(N)585 propulsion rotor was spun to 5000 rpm on both hydraulic-supports and rigid-supports. Curves in this report show transmitted force levels for each configuration and comparisons are made with calculated values. In order to complete the dynamics picture, plots giving the journal vibration are also included.

At 4500 rpm, transmitted force (measured in lbs.) dropped by a factor of 15 on hydraulic-supports when compared with rigid-supports. However, the level failed to go as low as predicted because of friction in the hydraulic-supports.

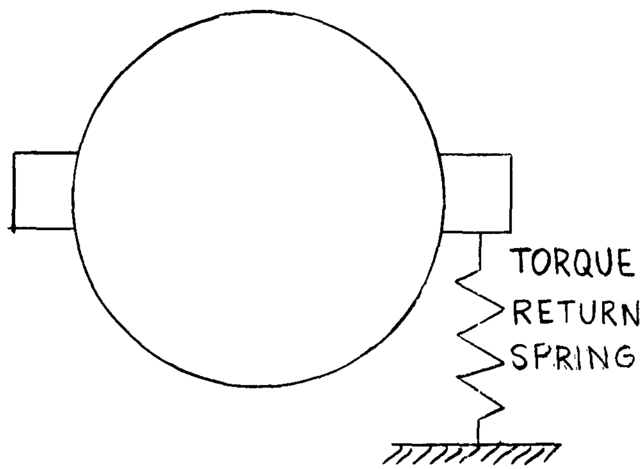
In contrast to the hydraulic-supports, a system incorporating mechanical springs (Ref.4) could be designed to overcome the friction disadvantage. Because of the establishment of profound noise reduction by a flexibly-supported bearing in this investigation and Ref. 4, it is recommended that a prototype be designed and applied to submarine high-speed machinery, but with mechanical springs rather than hydraulic-supports.

TEST APPARATUS

To determine the advantage of hydraulic-supports, two test build-ups were investigated: (1) rigid-supports and (2) hydraulic-supports.

Assembly drawings are included in Figs. 2 and 3, while Fig. 1 pictures the arrangement of the test components. A small steam turbine powered the test rotor up to 5000 rpm. The rotor, better viewed in Fig. 4, came from a Skipjack Class SS(N)585 submarine propulsion turbine.

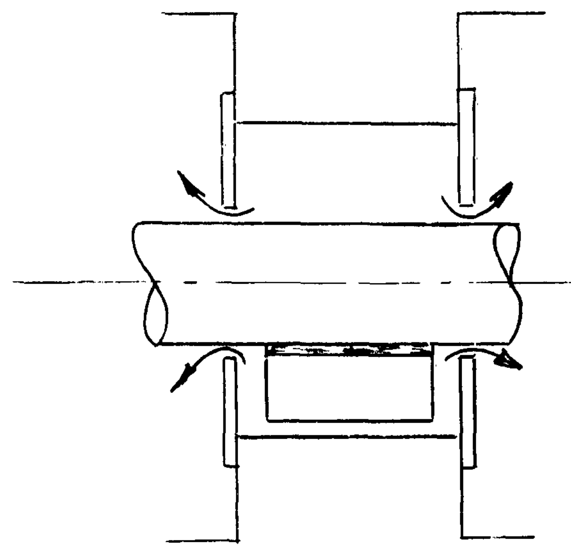
It is seen in Fig. 4 that care was taken not to interfere with the isolating function of the hydraulic-supports. For example, flexible hosing supplies and drains lubricating oil; a 30 in. long 1-1/4 in. diameter jackshaft connects the drive turbine; and a 35 lb/in. spring (see sketch) allows adjustment for bearing torque.

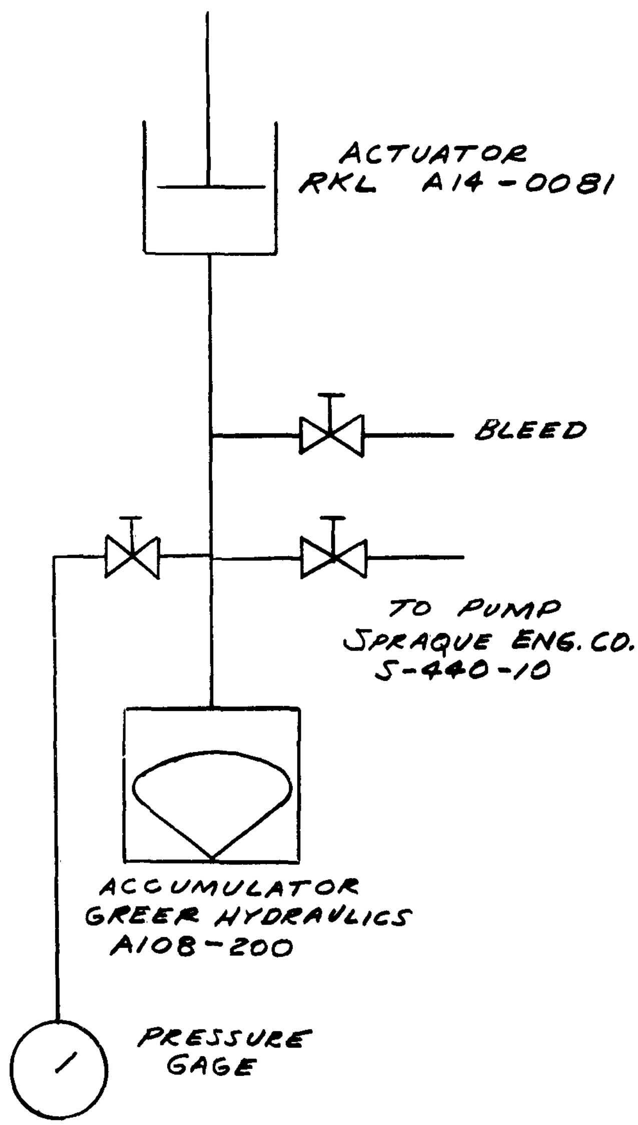


Since pivoted-shoe bearings are inherently stable, they were utilized to preclude the possibility of oil-whip due to the soft supports. A drawing of the shoes is reproduced in Fig. 5, while Fig. 6 is a photograph of a front and back view. Each 60° sector was supported on a spherical button so as to have rotational freedom in both the circumferential and axial directions. The axial feature permitted good alignment.

Instead of the equal spacing of three or more shoes around the circumference as in conventional practice, only two shoes cradled the rotor as shown in Fig. 7. This had the advantage of eliminating the intermediate preload effects of the upper pads, and simplified the force transmission path for a more accurate analytical description. Navy 2190TP oil (MIL-L-17331) lubricated the shoes, with the oil supply temperature maintained within $\pm$ 2°F with

Bailey Meter regulating equipment. The oil flooded the bearing cavity, with the flow orificed through the clearnace space of the end seals as illustrated in the sketch.





A schematic of the hydraulic system is shown in the adjacent diagram. Two such systems supported each bearing. The bearing housing was floated on the actuator pistons, with accumulators included in the hydraulic lines to provide the flexibility. Because of its low viscosity, kerosene was used as the hydraulic fluid to reduce damping effects. A Spraque diaphragm pump pressurized each system separately. The valve to the pump could be shut forming a closed system between the actuator and the Greer blatter-type accumulator.

The hydraulic actuators held the bearing housing as described in Fig. 8. Since the closed system application demanded zero leakage, a rolling diaphragm sealed the piston of the actuator, an RKL A14-0081 Rollomotor. The supports were located along mutually perpendicular axes and rested on Endevco force gages. The force transmitted to the bedplate was completely described by the transducers. The bearing housing sat on needle bearings to eliminate tangential forces. In order to reduce the friction to a very low level, provision was made to rotate the inner race of the antifriction bearing with gear-motors. However, this refinement was unnecessary when the transmitted force proved larger than that predicted.

Fig. 9 provides a close-up view of the hydraulic-support. The safety supports (shown hiding the contact between the needle bearing and the bearing housing) held the housing when the hydraulic-supports were inoperative. When the hydraulic system was pressurized, a gap of .040 in. was maintained between the safety member and the bearing housing.

In order to provide a standard for comparison, rigid-supports replaced hydraulic-supports in the second test configuration. The plate supports, seen in Fig. 10 and 11, simulated rigid-supports and yet allowed the force to be separated into X and Y components for meaningful measurements.

INSTRUMENTATION

Table I lists the instrumentation used during the tests and Fig. 12 shows the instrumentation arrangement.

Transducers, to monitor transmitted force and journal vibration, were concentrated at the bearing opposite the driven end of the test rotor. Both measurements were made along mutually perpendicular axes, 45° from the vertical.

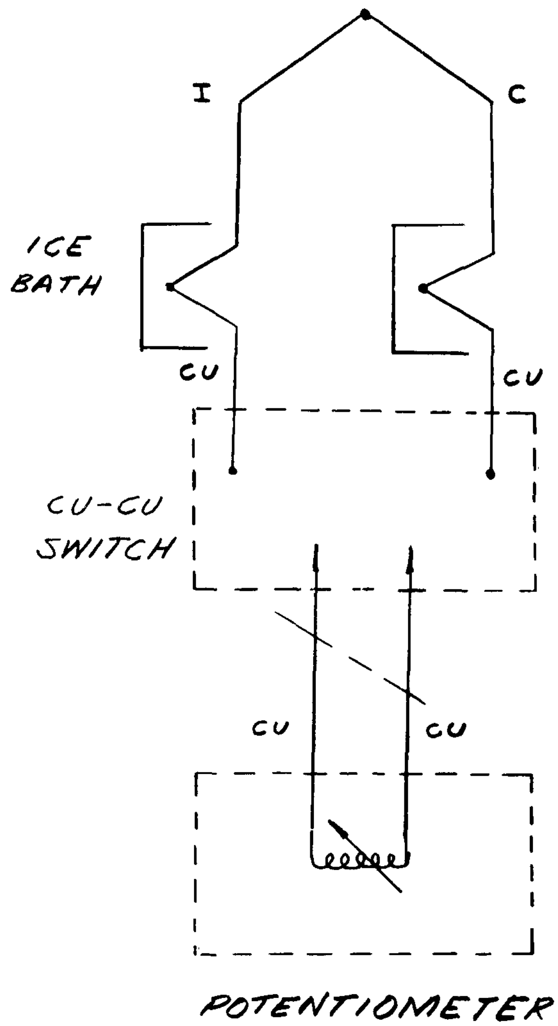
Model 2106 Endevco force gages, with a Model 2622 Endevco power supply and Model 2616B Endevco amplifiers, provided a voltage proportional to the transmitted force. The wave shape and level were determined with a Hewlet Packard oscilloscope, Model 122A. The accuracy of the oscilloscope was checked with a calibrated Model 400D Hewlet Packard voltmeter and found to be within the least count of its scale. The gages, which incorporated a crystal material to measure the dynamic force, had a stiffness of at least 2×10^7 lb/in. The sensitivities, which had been factory determined, were corrected for system capacitance and the mounting bolts.

Vibration was measured with Westinghouse HQ balancing instrumentation composed of (1) pick-up, (2) sine-wave generator, (3) amplifier, and (4) meter box. The sine-wave generator attached to the end of the test rotor providing a signal at the rotational frequency while pick-ups, which were in contact with the journal, produced an output proportional to velocity. The amplifier enabled a 1X, 3X, and 10X amplification of this signal. The pick-up and generator signals were multiplied in the meter box, acting as a wattmeter, to provide a read-out of the vibration level. By this means, only the operating speed component of vibration was displayed, and higher harmonic were eliminated since,

$$\int (\sin \omega t) \sin(n\omega t + \phi) = 0 \qquad n = 2, 3, 4, \cdots$$

The vibration equipment was calibrated as a system at various speeds using an eccentric cam.

An Electro-Products magnetic pick-up sensed speed from a 60 tooth gear and an E-put meter displayed the speed.



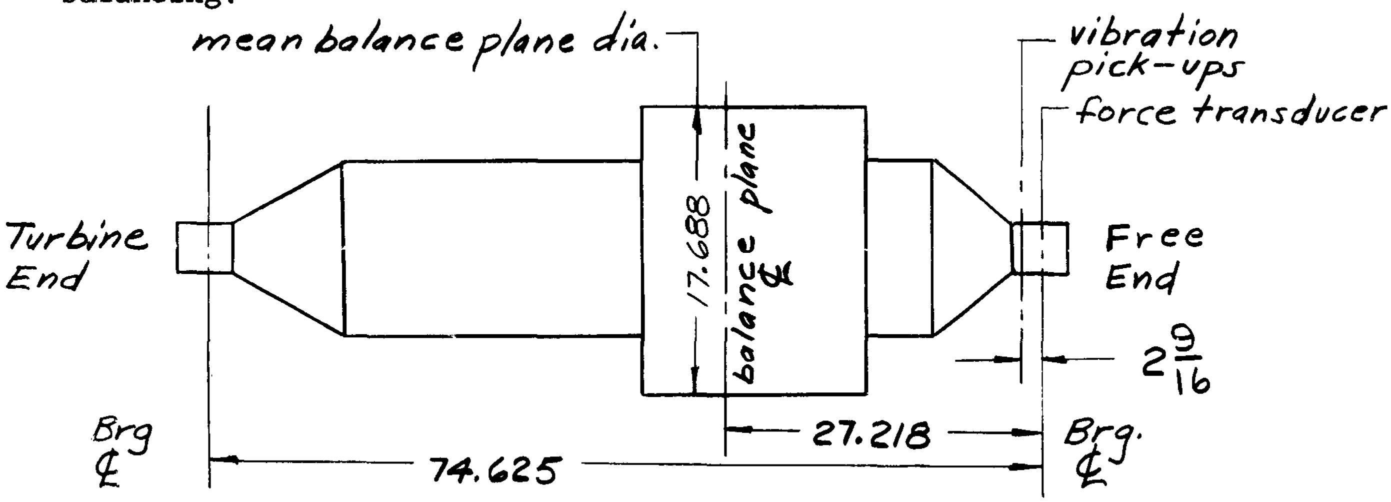
Spring contact, iron-constantan thermocouples, Fig. 5, sensed the babbit surface temperatures in the bearing pads. These temperatures were measured using a bridge-type Leeds Northrup potentiometer with the set-up shown on this page. All thermocouples had been calibrated in temperature baths.

Fischer Porter flowraters indicated the oil flow to each bearing.

TEST PROCEDURE

For the two test configurations: (1) rigid-supports and (2) hydraulic-supports, transmitted force and journal vibration data were collected for a known unbalance in the balance plane. The same unbalance was inserted 180° from its original location, and the test was repeated.* In addition, test data were taken with all inserted unbalance removed, leaving only residual unbalance. The values were combined, as described in the Calculation Section, to eliminate the effect of the residual. Two replicants were run of every test to guarantee repeatable results.

Before testing it was intended to balance the rotor; however, the residual unbalance proved sufficiently low to forgo balancing.

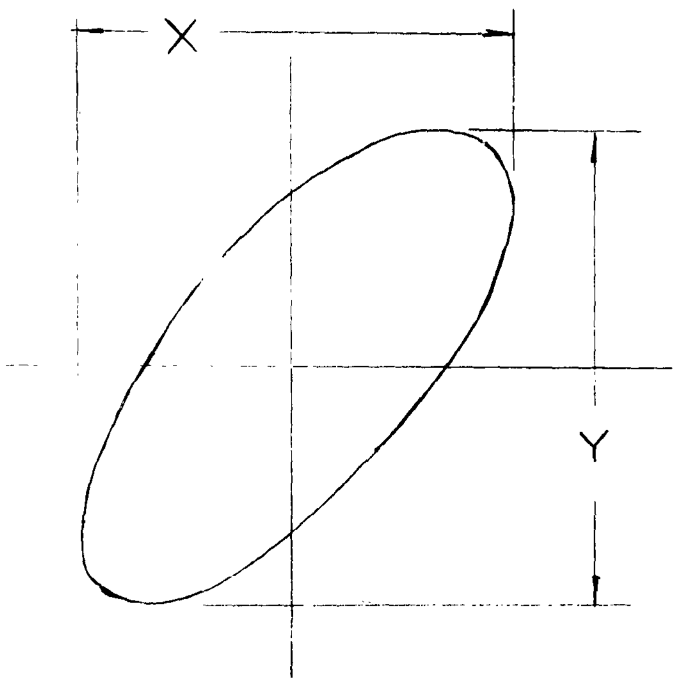


Sixteen, equally spaced, 1 inch threaded holes composed the balance circle, whose location is noted in the above drawing. Each balance plug weighed approximately 160 gm.

*-This refinement was dispensed with in the case of the hydraulic-supports because the residual unbalance was small compared with the accuracy of the experiment.

The table below lists the nominal conditions of the test:

	HYDRAULIC-SUPPORTS	RIGID-SUPPORTS
SPEED - rpm	1500, 2000, 3000, 4000, 5000	1500, 2000, 3000, 4000, 4500
OIL FLOW PER BRG - gpm	11	11
OIL SUPPLY TEMP. - °F	100	100
UNBALANCE - gm	160	64

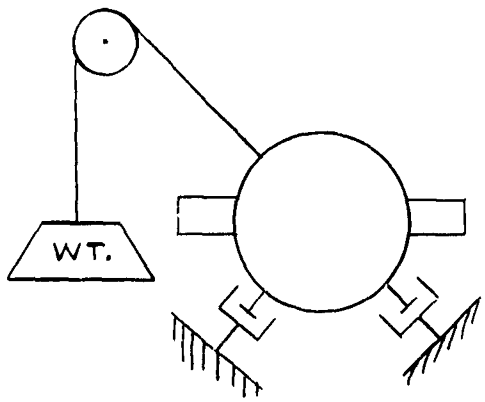


The signals from the two Endevco force gages, located at right angles as shown in Fig.8, were displayed on the corresponding X and Y-coordinates of the two channel oscilloscope. The peak-to-peak values of the transmitted force (see sketch) were recorded, although single rather than double amplitudes are presented in the curves to be discussed later.

Westinghouse HQ instrumentation measured journal vibration along the same axes as the force transducers. Again peak-to-peak values were recorded.

So as to maintain a consistent variation of the Sommerfeld number with speed, the bearing oil flow, supply temperature, drain temperature, and bearing housing pressure were monitored. Measuring the bearing pad temperature enabled calculation of an average viscosity.

Some measurements were characteristic only to the hydraulic-supports. For example, the precharge air pressures in the blatter-type accumulators were checked before and after each day's testing. If this value varied from the predetermined value, the accumulator was recharged before further testing. In addition, the hydraulic line pressure was recorded at each speed point.

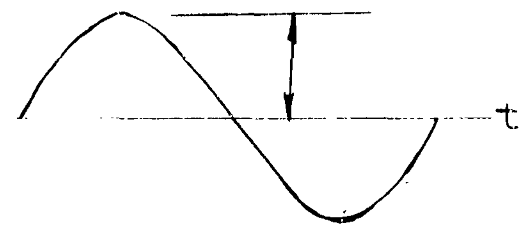


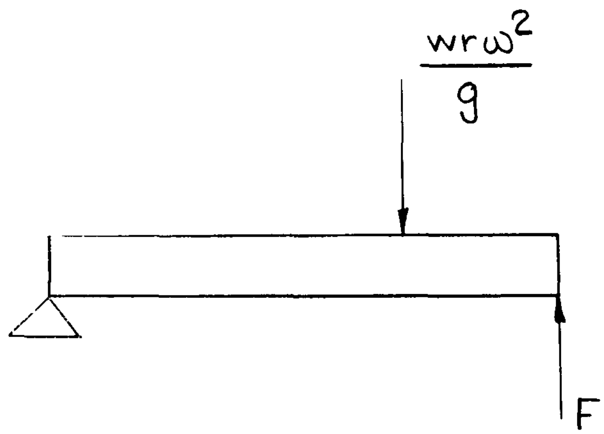
Before the rotating tests, the stiffness of each hydraulic-support was evaluated using dead weights (see sketch). The weights were increased to 160 lb. in 20 lb. increments. A dial indicator in line with the pull measured the bearing housing displacement. The pressure changes in the hydraulic lines were also recorded.

RESULTS

1. Rigid-Supports

Figs. 14 and 15 plot the experimental values of journal vibration and transmitted force produced by a unit unbalance at the balance plane. In Vol. 1 of this report, it was shown that the oil film properties are equal in the X and Y-directions and the force and vibration responses would theoretically be the same for the two directions. Therefore, the X and Y components of the test values were averaged for plotting in Figs. 14 and 15. Single amplitude values are given (see sketch).





For reference, the unattenuated force is included in Fig. 15. The unattenuated force is defined as the reaction at the bearing if the unbalance is applied as a non-rotating force. The experimental points fell close to the unattenuated curve although both the bearing oil-film and rotor have some flexibility. Resonance conditions appeared at approximately 2400 and 4200 rpm.

Table II summarizes the test data for each run. The calculated values of Sommerfeld number for both hydraulic and rigid-supports are presented in Fig. 13.

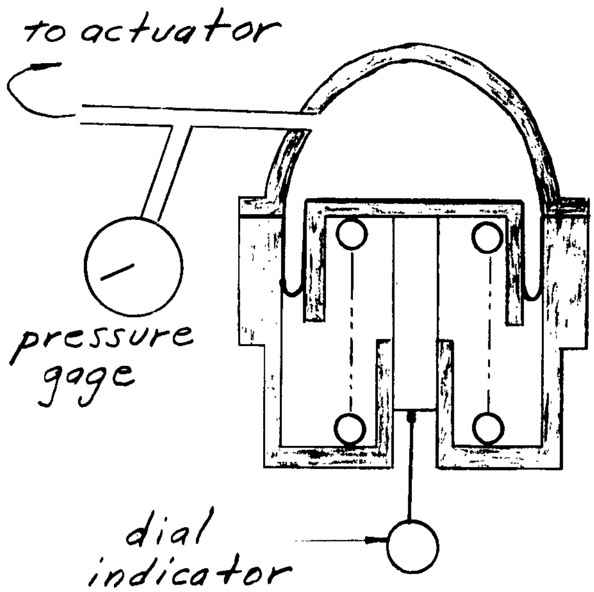
2. Hydraulic-Supports

For ease of comparison, the vibration and force curves are plotted in Figs. 14 and 15, together with the rigid-support curves. Although the journal vibration amplitudes are near the same values for the two cases, the transmitted force levels were appreciably less using hydraulic-supports. For the higher speeds, the force values level off and remain a constant magnitude while for the rigid-supports, the force continues to increase with speed.

In Fig. 16, the advantage of the hydraulic-supports is seen by plotting speed vs the force ratio. The force ratio is defined as the force transmitted with rigid-supports divided by that with hydraulic-supports. The most improvement was obtained where the force ratio was 15.

Table III presents a complete summary of the test data for hydraulic-supports.

Difficulties with friction in the hydraulic system appeared when the spring value was checked with dead weights, as described in the Test Procedure Section. The motion of the bearing housing was less than expected for the range of weights applied.



The spring and piston type accumulators* originally used were found to be the chief sources of friction. A rolling diaphragm or Bellofram sealed the piston for zero leakage. As weights were applied, the pressure gage responded (see sketch), although the dial indicator registered a large degree of sticking within the accumulator.

Greer blatter-type accumulators were substituted because of their inherent freedom from friction other than the small amount due to the elastic extension of the blatter.

By placing weights on the bearing housing and noting its displacement, the remaining friction was evaluated. The pressure in the hydraulic system was recorded so that the force change on the actuator piston could be compared to the weight added. As illustrated in Fig. 17, which presents typical results for one of the hydraulic-supports, a load of 160 lb. produced a force of 116 lb.

*-These were the same accumulators described in detail in Reference 4.

on the piston. This indicates static friction between mating parts preventing relative motion that might have occurred in the anti-friction bearings, grease lubricated bushings, actuator guides, or the actuator rolling-diaphragm. Fig. 18, however, indicates little remaining stiction in the accumulator. The spring constant of 4730 lb/in deduced from this curve agrees favorably with the calculated value of 5560 lb/in.

CALCULATIONS

1. Bearing Load

The bearing reactions R were determined from the rotor section properties given in Table IV by assuming that the bearings carried the weight of sections 1 to 17.

The bearing diameters D and the lengths L listed are measured values.

Turbine End

$L = 3\frac{19}{32} = 3.594$ in.

$D = 6.9837$ in.

$R = 2161.2$ lb.

$P = \frac{R}{LD} = 86.11 \frac{\text{lb.}}{\text{In}^2}$

Free End

$L = 3\frac{19}{32} = 3.594$ in.

$D = 6.9844$ in.

$R = 2293.2$ lb.

$P = 91.36 \frac{\text{lb.}}{\text{in}^2}$

2. Bearing Clearance

The clearance was found from the difference in micrometer measurements of the journal and bearing shoe diameters.

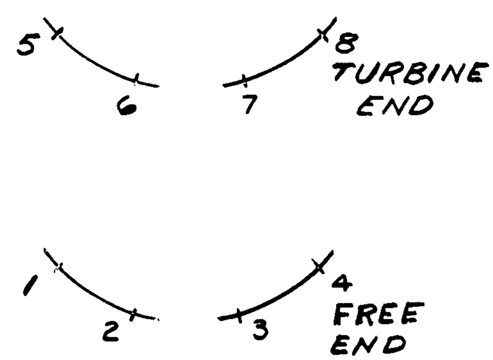
Turbine End: $C = \frac{6.9978 - 6.9837}{2} = .00705$ in.

Free End: $C = \frac{6.9978 - 6.9844}{2} = .0067$ in.

3. Viscosity

The viscosity-temperature relationship, Fig. 19, was determined for the lubricating oil from a laboratory sample.

For a test point, the effective viscosity was found from the average temperature on the bearing shoe surface. Location of the thermocouples is shown in the sketch.



LOOKING TOWARD TURBINE

3. Viscosity - Cont'd.

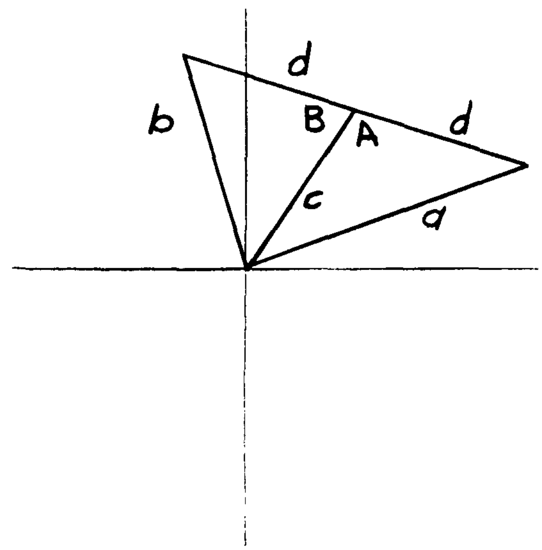
Turbine End: $T_{avg} = (T_5 + T_6 + T_7 + T_8) \div 4$

Free End: $T_{avg} = (T_1 + T_2 + T_3 + T_4) \div 4$

For the rigid-supports, each T_i represented an average of 6 values of temperature at the given speed (runs 21-50).

For the hydrualic-supports, each T_i was the average of 2 values (runs 1-10).

4. Transmitted Force



In finding the transmitted force for a given unbalance in the rotor, the residual was mathematically subtracted.

In the sketch, the vector c is the residual. The introduction of an unbalance weight produces a force a. With the weight placed 180° from its original location, b is transmitted. The formula for finding d, the component due only to the unbalance, is derived below by assuming a linear system.

$$b^2 = c^2 + d^2 - 2\,cd \cos B$$

$$a^2 = c^2 + d^2 - 2\,cd \cos A$$

$$A + B = 180°; \quad \cos A = -\cos B$$

$$b^2 = c^2 + d^2 - 2\,cd \cos B$$

$$\underline{a^2 = c^2 + d^2 + 2\,cd \cos B}$$

$$a^2 + b^2 = 2\,(c^2 + d^2)$$

$$d = \left(\frac{a^2 + b^2}{2} - c^2\right)^{1/2}$$

Transmitted Force - Cont'd.

example: rigid-supports

a		b		c	
run 21	154	run 41	169	run 31	84
run 26	159	run 46	179	run 36	84
average	156		174		84

$$d = \left(\frac{156^2 + 174^2}{2} - 84^2 \right)^{1/2} = 142 \text{ lb.}$$

For a 1.25 lb.-in. unbalance, this is the double amplitude of the force transmitted in the X-direction at 1500 rpm. However, Fig. 15 displays single amplitude values for a unit unbalance.

$$\text{transmitted force} = \frac{142}{(2)(1.25)} = 56.5 \frac{\text{lb.}}{\text{in.-lb.}}$$

In addition, the force in the Y-direction is averaged with that in the X-direction to give a data point in Fig. 15.

example: hydraulic-supports

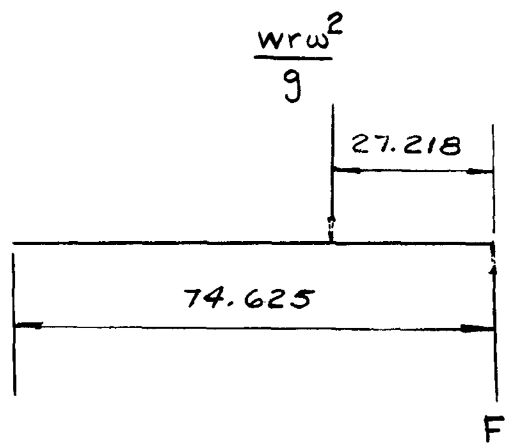
In the case of the hydraulic-supports, the residual proved small compared with the experimental error; therefore, it was not necessary to use the above procedure for subtracting the residual.

run 1	189
run 6	220
average	204 lb. (double amplitude in X-direction) per 3.11 in-lb.

$$\text{transmitted force} = \frac{204}{(2)(3.11)} = 32.8 \frac{\text{lb.}}{\text{in-lb.}}$$

This force was averaged with that in the Y-direction.

Transmitted Force - Cont'd.



For comparison, it was desirable to calculate the unattenuated force, defined as the bearing reaction due to a static force equal to the unbalance.

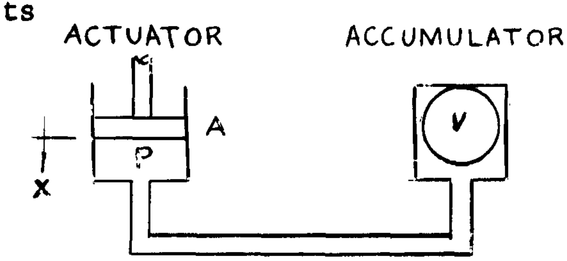
$$F = \frac{wr\omega^2}{g} \frac{(27.218)}{(74.625)}$$

For wr = 1.0 lb-in., the unattenuated force is plotted in Fig. 15 as a function of speed.

5. Vibration Displacement

The calculations of journal vibration duplicated those for the transmitted force.

6. Spring Value of Hydraulic-Supports



The spring value of the hydraulic-supports was derived as shown below:

$$F = pA$$

$$\Delta V = - A\Delta x$$

$$p = \frac{C}{V^k}$$

$$\text{But, } K_p = \frac{dF}{dx} = \left(\frac{dp}{dV} \frac{dV}{dx} \frac{dF}{dp} \right)$$

6. Spring Value of Hydraulic-Supports - Cont'd.

$$K_p = \frac{k\,A^2\,p}{V}$$

The compressed air volume V is determined from the isothermal relation,

$$V = \frac{po\ Vo}{P}$$

$$K_p = \frac{k\,A^2\,p^2}{poVo}$$

k = 1.4 (air)

A (actuator piston area) = 14.2 in^2

V_o (accumulator capacity) = 30.8 in^3

The accumulator charge pressure was measured before and after each series of testing over the speed range. p_o is the average of data taken during runs 1-10, with the X and Y-axes considered together.

p represents the average of the hydraulic line measurements made in runs 1-10 and in both X and Y-directions.

An average spring value was calculated for each bearing:

Free End:

$$K_p = \frac{(1.4)(14.2)^2(166.9)^2}{(46.2)\ (30.8)} = 5510 \text{ lb/in.}$$

Turbine End:

$$K_p = \frac{(1.4)(14.2)^2(159.8)^2}{(43.7)\ (30.8)} = 5360 \text{ lb/in.}$$

ANALYTICAL COMPARISON

The experimental results were compared to analytical ones, which were determined using the mathematical tools developed under this contract.

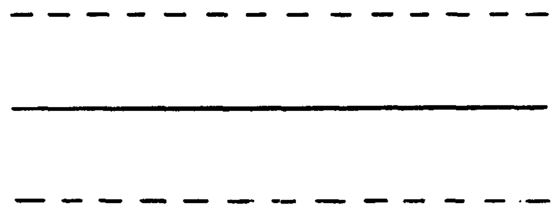
The generalized-rotor program, outlined in Part I of this volume, allowed the inclusion of the flexibility and damping of the support together with the spring and damping properties of the bearings. The analysis in Vol. 1 of this report enabled the determination of the bearing properties from the calculated values of the Sommerfeld numbers given in Fig. 13. The spring and damping values, presented in Table V, are equal in the X and Y-directions because of the symmetry of the bearing geometry.

The rotor section data are included in Table IV. Other information used in the generalized-rotor program is given below:

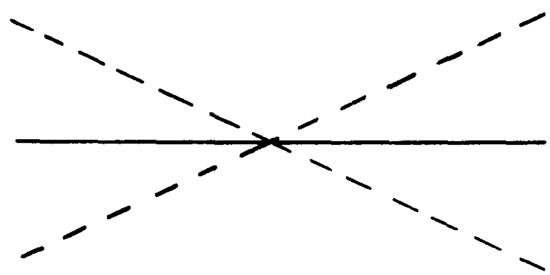
Bearing housing weight	(free end)	=	493 lb.
Bearing housing weight	(turbine end)	=	517 lb.
Support spring value	(free end)	=	5510 lb/in.
Support spring value	(turbine end)	=	5360 lb/in.
Modulus of elasticity		=	30×10^6 psi

For the rigid-supports, the calculated journal response is displayed in Fig. 20 together with the experimental points for comparison. The theoretical curve shows no peaking within the speed range. This contrasts with the experimental data in which peaks appear at 2200 rpm and 4200 rpm. Calculations showed that these resonant peaks were due to the flexibility of the support structure of between 10^5 and 5×10^5 lb/in., although a flexibility of 10^7 lb/in was calculated from the physical dimensions of the support plates. When the transmitted force is compared in Fig. 21, better agreement is seen between the calculated and experimental values.

For hydraulic-supports, the analytical correlation of the journal response data is presented in Fig. 22. The test points indicate a resonance condition at a higher speed than predicted. This is the result of the friction remaining in the hydraulic-supports. However, despite the friction, agreement is fairly good. At the higher speeds, both curves level out to a constant vibration amplitude indicating the tendency of a rotor to spin about an axis having the least moment of inertia. The calculated curve shows resonant peaks at 260 and 440 rpm which represent the translational and conical modes respectively (see sketch).



TRANSLATIONAL MODE



CONICAL MODE

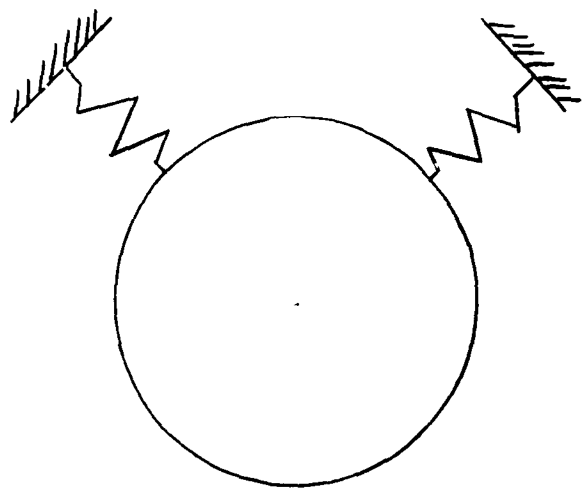
Fig. 23 compares the transmitted force levels for the hydraulic supports. The difference is again attributed to friction in the supports. Calculations assuming a damping value of 50 lb-sec/in. are included to show the effect of even a modest amount of damping on the transmitted force. That is, the resonant peaks occur at higher speeds, and the force level increases beyond the criticals. It is also interesting to observe that this damping drastically reduces the resonance peaks.

DISCUSSION

The test verified the analytical results that in general flexible-supports could produce a sizeable reduction in transmitted force. Moreover, it substantiated that hydraulic-supports were a feasible approach for obtaining this flexibility.

Successful operation of the test rotor up to 5000 rpm yielded no unusual experiences or overwhelming difficulties. For example, there were no problems with instabilities or those introduced by resonances of subcomponents. Although provisions were made to lock-out the hydraulic-supports and to introduce additional damping so that the rotor could pass through the critical speeds, these precautions proved unnecessary.

The main failing of the investigation was the inability to accurately predict the force reduction obtainable with hydraulic-supports since friction appeared to be inherent in the supports. However, the force reduction advantage of flexible-supports makes it appealing for submarine application. They could be applied to propulsion turbines, turbine-generator sets, and other rotating equipment. For a ship's prototype design, an overhead coil-spring arrangement (see sketch and Ref. 4) is recommended, since emphasis should be placed on limiting friction for further force reduction and more accurate predictability. Mechanical spring supports could be designed with little rubbing between mating parts.



In order to save space, it is further advised that a partial bearing be substituted for the pivoted-shoes since no oil-whip instability problem arose with its application in Ref. 4. If instability occurred in an isolated case, damping could easily be added to the supports.

In most applications, a servo-return system, such as developed in Ref. 4, would be incorporated into the supports to correct for force changes that would produce excessive deflections of the rotor. This system could be designed with sufficient ruggedness to insure reliability. Moreover, even complete failure would not interfere with the normal machinery operation.

RECOMMENDATIONS

1. Apply a prototype of the flexible-supports to a high-speed rotor onboard a submarine.

2. Substitute mechanical springs for the hydraulic-supports. Mechanical springs are a simplier scheme that would reduce friction sources.

3. Include an automatic positioning system if the deflections on the soft supports prove excessive.

4. Substitute a partial bearing for the pivoted-shoe bearing in order to save space in an area that will be crammed with the flexible-support hardware.

5. Analyze the shock problem in designing the ship's prototype for possible inclusion of snubbers.

6. Redesign the hydraulic-supports if included in a future application. In order to reduce friction, it is advised that an overhead design with flat diaphragm actuators be considered.

7. Defer the testing of (1) a cylindrical bearing and/or (2) elastomer-supports. In making the proposal for the present contract, it had been advised that the desirability of investigating the above items would be determined at the conclusion of the present tests. Now, little advantage is seen in this extension before the application of the present knowledge to a prototype design.

CONCLUSIONS

1. Hydraulic-supports proved to be a feasible way of isolating a rotor, reducing the force transmitted to the bedplate by as much as a factor of 15 when compared with rigid-supports.

2. No instability condition arose when the rotor operated to 5000 rpm.

3. There was no difficulty in passing through the criticals that were lowered by the soft supports.

4. Friction in the hydraulic-supports prevented accurate agreement with calculated levels of the transmitted force.

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4. J. C. Fistere, Jr. and W. H. Dickson, "Report on the Feasibility of Flexibly Supported Bearings for Submarine Propulsion Turbines", Bureau of Ships Contract NObs 78931, Westinghouse Electric Corporation, September 1964.

TABLE I

INSTRUMENTATION LIST

Instrument	Make	Model	Serial No.	Range	Least Count	Purpose
HQ Pick-up	Westinghouse		IY-P-7CA			Journal vibr.
HQ Pick-up	Westinghouse		IY-P-7CB			Journal vibr.
HQ Meter Box	Westinghouse		IY-B-25			Journal vibr.
HQ Generator	Westinghouse		IY-G-6A			Journal vibr.
HQ Amplifier	Westinghouse					Journal vibr.
Force Gage	Endevco	2106	CA46			Force
Force Gage	Endevco	2106	CA47			Force
Power Supply	Endevco	2622	IY-P-25A			Force
Amplifier	Endevco	2616B	IY-A-16S			Force
Oscilloscope	Hewlet Packard	122A	IY-O-4V	0-10v/cm		Force
VTVM	Hewlet Packard	400D	IY-V-19H	0-300v		Force
Audio Oscillator	Hewlet Packard	200CD	IY-O-2AG	5-600,000 cps		Force
Flowrater	Fischer Porter	8-35-600/7D	W8-1170/1	0-600 mm	1 mm	Brg. oil flow
Flowrater	Fischer Porter	8-35-600/7D	T1-1409/7	0-600 mm	1 mm	Brg. oil flow
Hg-in-glass therm.	Phila. Thermo.			25-145°F	5°F	Brg. inlet temp.
Hg-in-glass therm.	Phila. Thermo.			30-240°F	2°F	Brg. drain temp.
Hg-in-glass therm.	Phila. Thermo.			30-24o°F	2°F	Brg. drain temp.
Potentiometer	Leeds Northrup		IY-P-16R			Brg. pad temp.
Bourdon press.gage	Ashcroft			0-30 psig	.5 psi	Brg. hsg. press.
Bourdon press.gage	Ashcroft			0-30 psig	.5 psi	Brg. hsg. press.
Bourdon press. gage	Ashcroft		TG-077	0-160 psig	1.0 psi	Hyd. line press.
Bourdon press. gage	Ashcroft		EL-160	0-160 psig	1.0 psi	Hyd. line press.
Bourdon press. gage	Ashcroft		EL-162	0-160 psig	1.0 psi	Hyd. line press.
Bourdon press. gage	Ashcroft		EL-161	0-160 psig	1.0 psi	Hyd. line press.
Digital counter	Beckman					Rotor speed

TABLE II

DATA SUMMARY - RIGID SUPPORTS

Run		21	22	23	24	25	26	27	28	29	30
Speed - rpm		1500	2000	3000	4000	4500	1500	2000	3000	4000	4500
Force - lb. peak to peak											
X-axis*		154.3	384.	584.	1840.	1740.	159.	369.	613.	1740.	1638.
Y-axis		77.7	350.	350.	680.	680.	82.6	320.	350.	682.	632.
Vibration- mils peak to peak											
X-axis		.29	1.43	.24	1.68	1.75	.24	1.18	.48	1.61	1.78
Y-axis		.29	1.58	.99	1.60	1.58	.28	1.48	1.13	1.64	1.53
Brg. pad temp.-°F	T1	141.0	147.6	158.7	167.0	170.4	141.8	148.1	158.0	166.6	170.5
	T2	125.8	129.0	134.9	138.2	140.2	126.6	129.5	133.8	136.3	140.2
	T3	136.1	139.7	146.3	156.3	159.2	137.1	139.8	145.7	154.8	159.2
	T4	123.0	123.2	125.3	130.5	133.2	121.6	121.9	124.3	130.5	133.2
	T5	140.2	146.6	160.4	172.6	177.6	140.6	147.2	157.2	172.1	178.2
	T6	124.9	127.7	135.0	140.2	142.3	125.5	128.0	133.6	140.0	142.5
	T7	135.7	140.2	152.5	162.6	166.6	136.2	140.5	151.5	162.2	166.7
	T8	122.0	123.1	128.5	133.3	135.6	122.7	123.5	127.2	133.1	136.1
Brg. housing press.- psig											
free end		7.2	7.2	5.4	4.5	4.0	7.0	6.2	5.1	4.2	3.8
turbine end		7.8	7.2	5.8	5.0	5.0	7.5	7.2	6.0	5.2	5.0
Brg. Oil flow - gpm											
free end		12.4	12.5	12.9	13.0	12.8	12.4	12.3	12.4	12.5	12.4
turbine end		12.3	12.3	12.2	12.2	12.7	12.3	12.3	12.4	12.4	12.4
Brg. oil supply temp.- °F		104.	104.	105.	106.	106.	104.	104.	105.	106.	106.
Brg. oil drain temp. - °F											
free end		112.	114.	120.	125.	127.	113.	114.	119.	126.	128.
turbine end		112.	113.	119.	125.	127.	112.	113.	118.	126.	128.
Unbalance - grams		64.2 @ #1 hole +					64.2 @ #1 hole				

*-See Fig. 8

+-16 equal spaced holes

TABLE II (Continued)

Run		31	32	33	34	35	36	37	38	39	40
Speed - rpm		1500	2000	3000	4000	4500	1500	2000	3000	4000	4500
Force - lb. peak to peak											
X-axis		84.3	215.	306.	1012.	982.	84.4	236.	276.	952.	920.
Y-axis		43.7	170.	155.	422.	422.	40.8	165.	170.	379.	379.
Vibration - mils peak to peak											
X-axis		.21	.81	.25	1.00	1.02	.17	.75	.21	1.05	1.02
Y-axis		.17	.90	.54	.84	.90	.13	.89	.59	.84	.82
Brg. pad temp. - °F	T1	141.6	148.2	157.5	167.1	169.3	139.8	147.1	156.9	166.5	170.2
	T2	126.1	129.2	133.1	138.7	138.9	124.0	127.9	132.4	138.5	140.4
	T3	136.4	139.7	144.5	154.5	157.2	134.2	138.6	143.7	154.0	158.9
	T4	121.3	121.5	123.2	130.5	130.5	119.0	120.3	122.0	130.5	132.8
	T5	139.7	146.7	158.4	171.3	175.8	138.5	145.8	156.9	172.9	177.7
	T6	125.1	128.0	132.5	139.7	140.3	123.0	126.6	131.3	140.2	142.5
	T7	136.0	140.2	150.0	160.9	164.7	133.7	138.9	148.1	161.7	166.2
	T8	122.2	123.1	125.6	133.1	133.6	120.1	121.7	124.4	133.2	135.6
Brg. housing press.- psig											
free end		7.0	6.5	5.2	4.0	3.9	7.5	6.7	5.2	4.1	3.8
turbine end		8.2	7.8	6.4	5.3	5.2	8.8	8.0	5.6	5.6	5.2
Brg. oil flow - gpm											
free end		12.3	12.3	12.4	12.4	12.3	12.5	12.4	12.4	12.4	12.4
turbine end		12.3	12.4	12.4	12.5	12.4	12.3	12.4	12.4	12.4	12.4
Brg. oil supply temp.- °F		106.	105.	105.	108.	106.	103.	104.	105.	105.	106.
Brg. oil drain temp. - °F											
free end		113.	114.	118.	126.	127.	109.	113.	117.	124.	127.
turbine end		112.	113.	117.	126.	127.	111.	112.	116.	123.	128.
Unbalance - grams			residual	only				residual	only		

TABLE II (Continued)

	41	42	43	44	45	46	47	48	49	50
Run	41	42	43	44	45	46	47	48	49	50
Speed - rpm	1500	2000	3000	4000	4500	1500	2000	3000	4000	4500
Force - lb. peak to peak										
X-axis	169.	384.	553.	1790.	1740.	179.	400.	568.	1790.	1740.
Y-axis	77.8	335.	320.	680.	778.	82.6	320.	320.	632.	730.
Vibration - mils peak to peak										
X-axis	.36	1.64	.48	1.74	1.77	.36	1.60	.48	1.81	1.77
Y-axis	.35	1.78	1.15	1.74	1.82	.34	1.73	1.22	1.78	1.79
Brg. pad temp. - °F T1	140.9	147.3	157.0	167.2	168.3	140.0	147.3	157.0	166.0	169.0
T2	125.5	128.5	132.2	138.5	137.2	124.4	128.5	132.5	137.3	138.3
T3	135.9	139.1	144.0	154.7	155.9	134.8	139.0	143.9	153.5	157.0
T4	120.7	120.6	122.3	130.3	129.0	119.5	120.9	122.4	129.4	130.2
T5	139.8	146.4	157.8	163.6	175.7	139.0	146.3	157.0	171.0	175.3
T6	124.4	127.2	131.8	139.8	139.2	123.4	127.5	132.0	139.1	140.2
T7	135.3	139.7	148.8	160.2	163.7	134.2	139.7	148.0	160.0	163.0
T8	121.6	122.2	125.1	132.7	132.0	120.2	122.0	124.6	132.2	133.0
Brg. housing press. - psig										
free end	7.2	6.5	5.3	4.1	3.8	7.3	6.5	5.2	4.1	4.0
turbine end	8.6	8.0	6.8	5.6	5.4	9.0	8.0	6.8	5.8	5.5
Brg. oil flow - gpm										
free end	12.4	12.3	12.4	12.4	12.3	12.4	12.5	12.4	12.4	12.4
turbine end	12.3	12.4	12.4	12.4	12.4	12.3	12.4	12.3	12.5	12.4
Brg. oil supply temp. - °F	105.	104.	104.	107.	103.	103.	104.	105.	107.	103.
Brg. oil drain temp. - °F										
free end	112.	113.	117.	125.	124.	111.	113.	117.	125.	126.
turbine end	111.	112.	116.	125.	124.	110.	112.	116.	125.	126.
Unbalance - grams	64.2 @ #9 hole					64.2 @ #9 hole				

TABLE III

DATA SUMMARY - HYDRAULIC SUPPORTS

Run		1	2	3	4	5	6	10	7	8	9
Speed - rpm		1500	2000	3000	4000	5000	1500	2000	3000	4000	5000
Force - lb. peak to peak											
X-axis*		189.	46.	224.	194.	189.	220.	40.9	204.	204.	184.
Y-axis		316.	180.	218.	155.	209.	359.	107.	224.	180.	194.
Vibration - mils peak to peak											
X-axis		1.05	.57	2.31	1.74	1.77	1.07	.51	2.00	1.59	1.54
Y-axis		1.07	1.15	2.18	1.59	1.68	1.16	.87	2.18	1.66	1.54
Brg. pad temp.- °F	T1	137.	143.4	152.4	159.2	167.0	133.2	140.5	150.5	158.5	165.8
	T2	120.	124.2	128.2	133.0	139.5	116.2	120.5	125.7	132.	139.6
	T3	130.6	134.7	136.5	145.8	157.6	127.2	131.6	137.6	146.	158.3
	T4	111.6	115.4	116.2	122.6	133.6	110.2	111.7	114.8	122.7	133.7
	T5	136.	142.4	151.5	157.8	167.3	133.4	141.2	151.6	160.	168.6
	T6	119.5	121.2	126.5	130.5	138.5	116.0	120.3	125.8	132.5	140.0
	T7	132.3	136.8	142.6	149.1	153.1	128.2	134.0	141.4	149.8	161.2
	T8	110.0	111.2	111.8	115.7	125.7	110.0	114.2	116.9	125.0	134.6
Avg. hydraulic press. -psig											
free end		152.1					152.4				
turbine end		145.7					144.5				
Avg. accum.charge press. -psig											
free end		32.					31.				
turbine end		29.					29.				
Brg. housing press.- psig											
free end		7.5	7.0	5.8	3.5	1.8	7.9	7.6	5.2	3.5	2.6
turbine end		7.2	6.7	5.2	4.4	4.0	8.2	7.2	5.2	4.0	4.0
Brg. oil flow - gpm											
free end		10.5	10.6	10.6	10.4	11.0	10.4	10.2	10.4	10.4	11.7
turbine end		10.3	10.4	10.3	10.9	12.6	10.4	11.3	10.3	10.4	12.2
Brg. oil supply temp.- °F		100.	100.	98.	100.	105.	98.	98.	99.	102.	105.
Brg. oil drain temp. - °F											
free end		105.	105.	109.	117.	130.	102.	105.	108.	118.	129.
turbine end		106.	109.	112.	120.	129.	104.	106.	112.	121.	131.
Unbalance - grams		159.5	@ #9 hole+				159.5	@ #9 hole			

* See Fig. 8

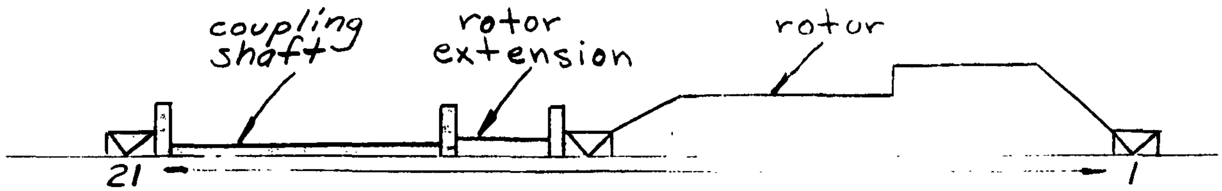
\+ 16 equal spaced holes

TABLE III (Continued)

Run		11	12	13	14	15
Speed - rpm		1500	2000	3000	4000	5000
Force - lb. peak to peak						
X-axis		23.0	2.3	20.4	21.5	17.9
Y-axis		32.1	5.3	32.1	24.3	19.5
Vibration - mils peak to peak						
X-axis		.17	.11	.52	.37	.38
Y-axis		.25	.07	.31	.18	.23
Brg. pad temp. - °F	T1	136.9	144.2	156.3	160.7	167.6
	T2	120.7	124.7	132.8	134.5	143.3
	T3	131.2	135.8	143.8	147.8	158.2
	T4	115.9	116.7	122.2	125.	133.7
	T5	136.4	144.0	156.2	161.0	168.3
	T6	120.3	124.2	131.8	134.	139.0
	T7	132.2	136.3	147.3	152.6	162.1
	T8	117.8	119.3	124.3	127.8	135.2
Avg. hydraulic press.- psig						
free end		150.6				
turbine end		143.4				
Avg. accum. charge press.-psig						
free end		31.				
turbine end		39.				
Brg. housing press.- psig						
free end		6.8	6.5	4.5	3.5	2.8
turbine end		6.8	6.0	4.7	3.8	4.0
Brg. oil flow - gpm						
free end		10.3	10.4	10.4	10.4	11.6
turbine end		10.3	10.3	10.4	10.4	12.4
Brg. oil supply temp.-°F		100.	98.	102.	100.	103.
Brg. oil drain temp. -°F						
free end		105.	106.	114.	118.	127.
turbine end		108.	108.	117.	122.	130.
Unbalance - grams		Residual only				

TABLE IV

ROTOR SECTION PROPERTIES



Station No.	Weight Lb.	Length In.	Cross Section I In^4
1	30.0	5.31	138.8
2	125.0	6.62	554.0
3	124.0	2.25	1102.0
4	382.0	5.91	3115.0
5	876.0	7.128	7580.0
6	0	2.652	7580.0
7	777.0	7.19	3350.0
8	570.0	8.57	1554.0
9	453.0	7.19	1508.0
10	235.0	5.00	1262.0
11	300.0	6.62	2130.0
12	313.0	4.87	543.0
13	82.0	2.13	188.0
14	32.0	3.19	117.8
15	36.72	7.42	20.0
16	28.21	8.65	5.476
17	90.43	12.375	.312
18	4.297	12.375	.12
19	4.297	12.375	.312
20	78.52	4.75	2.0
21	10.20	0	1.0

Bearing Stations	Unbalance Station
1 15 21	6

TABLE V

DAMPING AND SPRING VALUES OF BEARING OIL-FILM

	Speed rpm	Free End $10^5\bar{K}$ lb/in	Free End $10^3\bar{C}$ lb-sec/in	Turbine End $10^5\bar{K}$ lb/in	Turbine End $10^3\bar{C}$ lb-sec/in
	200	20.5	49.0	18.4	43.9
	1000	10.3	9.8	9.2	8.78
Rigid	2000	9.58	4.9	8.43	4.39
Support	3000	8.90	3.27	7.97	2.93
	4000	7.87	2.53	7.66	2.19
	5000	7.87	2.03	7.20	1.81
	6000	7.53	1.69	7.05	1.51
	200	17.8	49.0	15.9	43.9
	1000	9.92	9.8	8.89	8.78
	2000	8.90	4.9	8.28	4.39
Hydraulic	3000	7.87	3.38	7.20	3.02
Support	4000	7.53	2.53	6.90	2.27
	5000	7.53	2.03	6.74	1.81
	6000	7.53	1.60	6.74	1.5

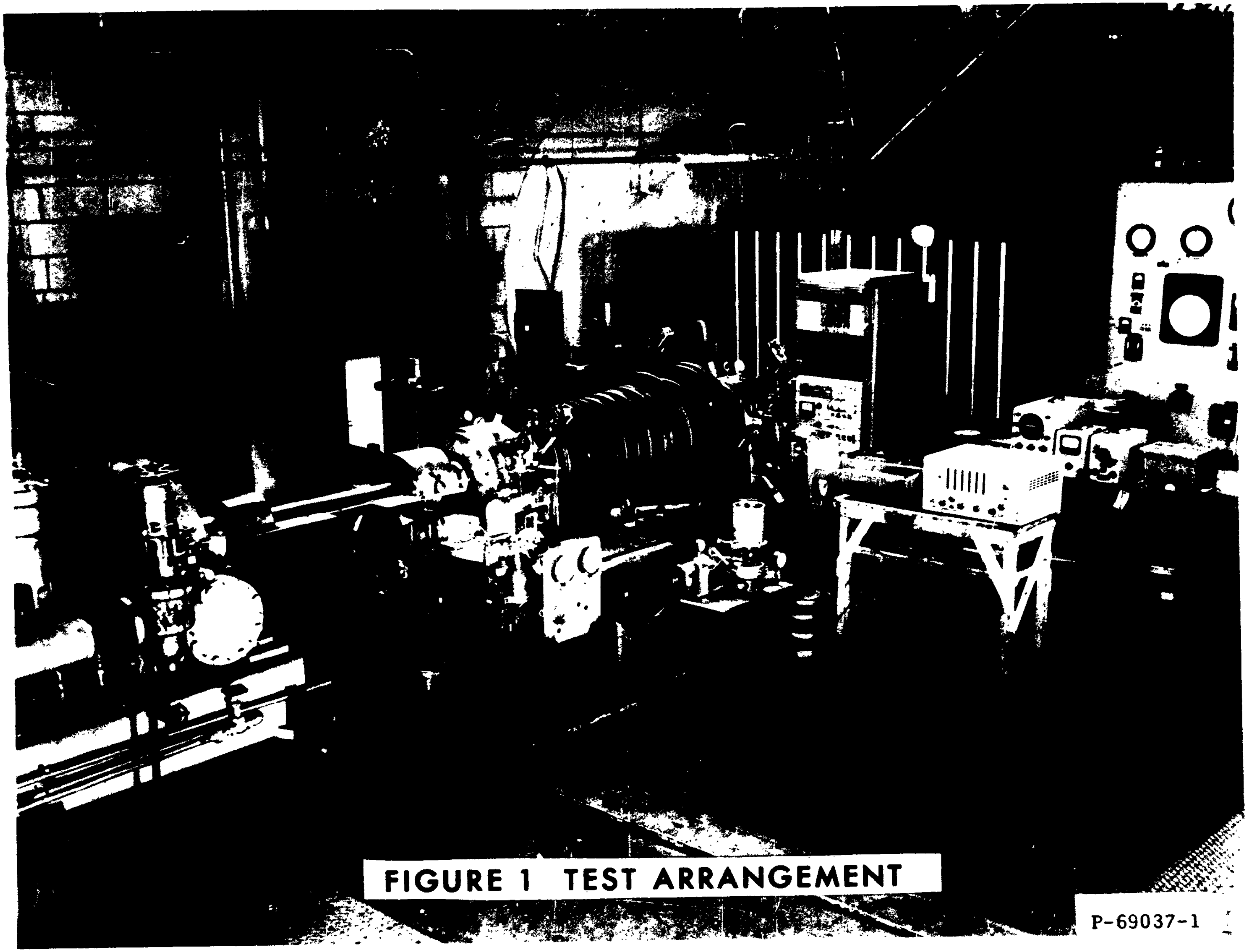


FIGURE 1 TEST ARRANGEMENT

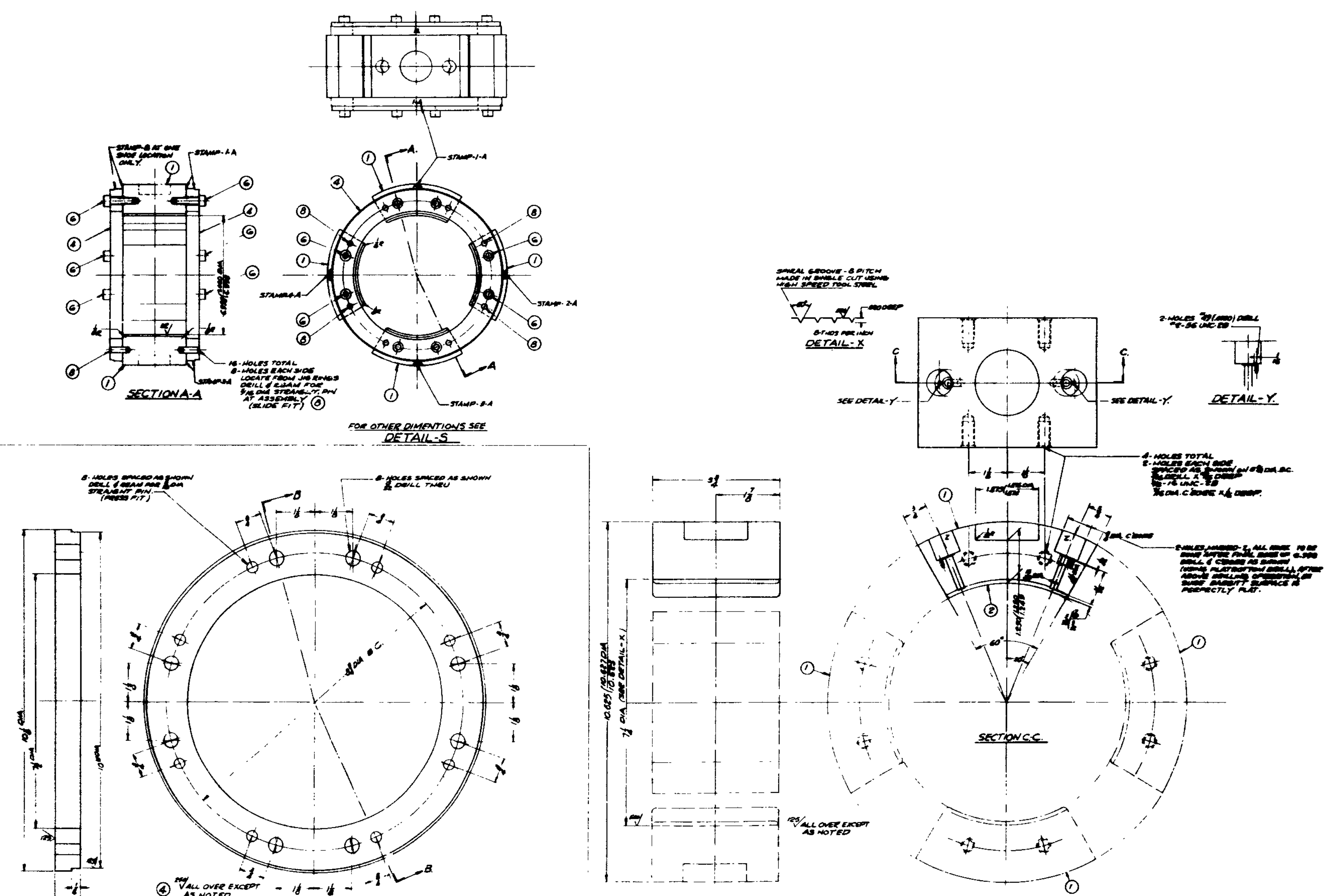


FIGURE 5 BEARING PADS

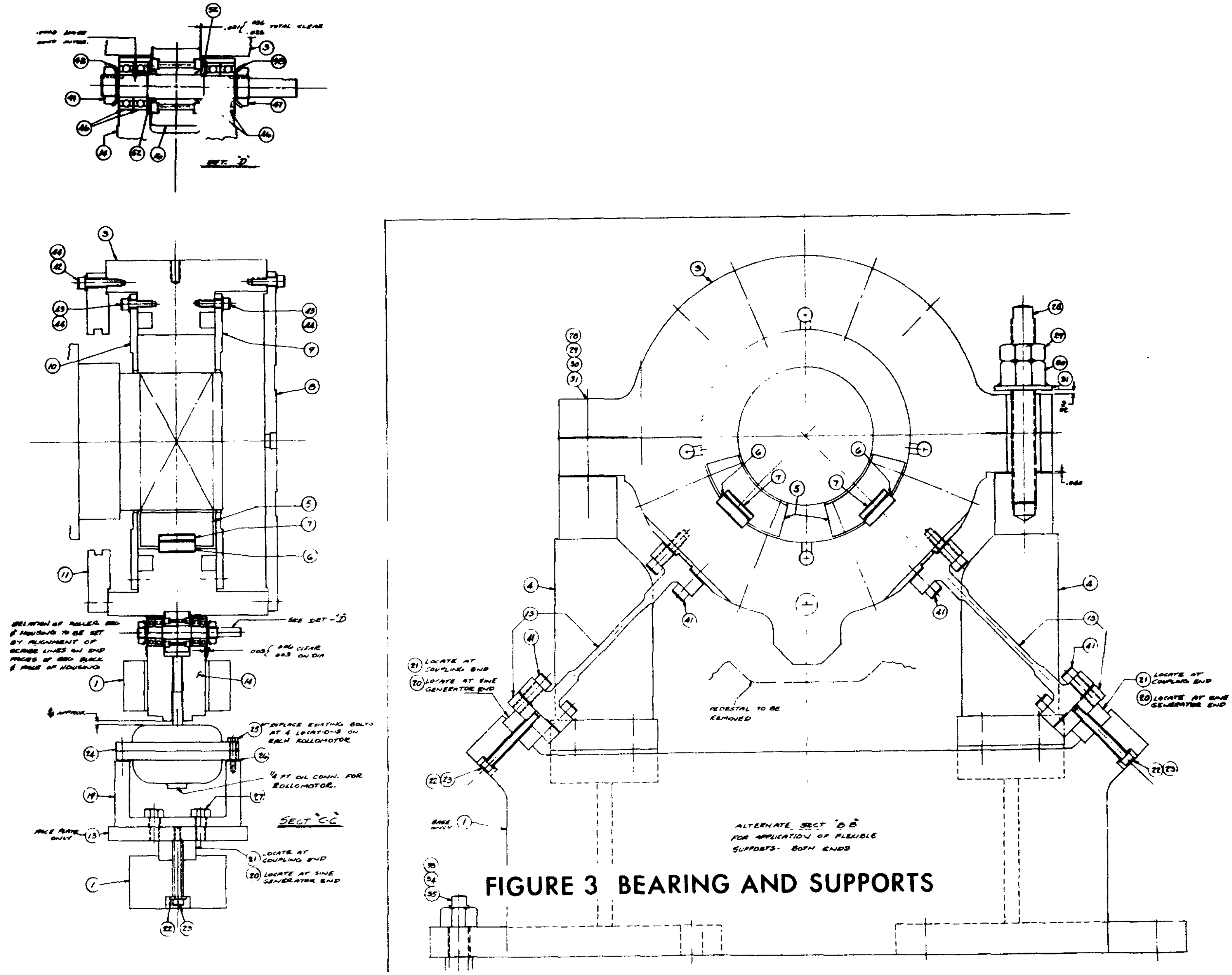


FIGURE 3 BEARING AND SUPPORTS

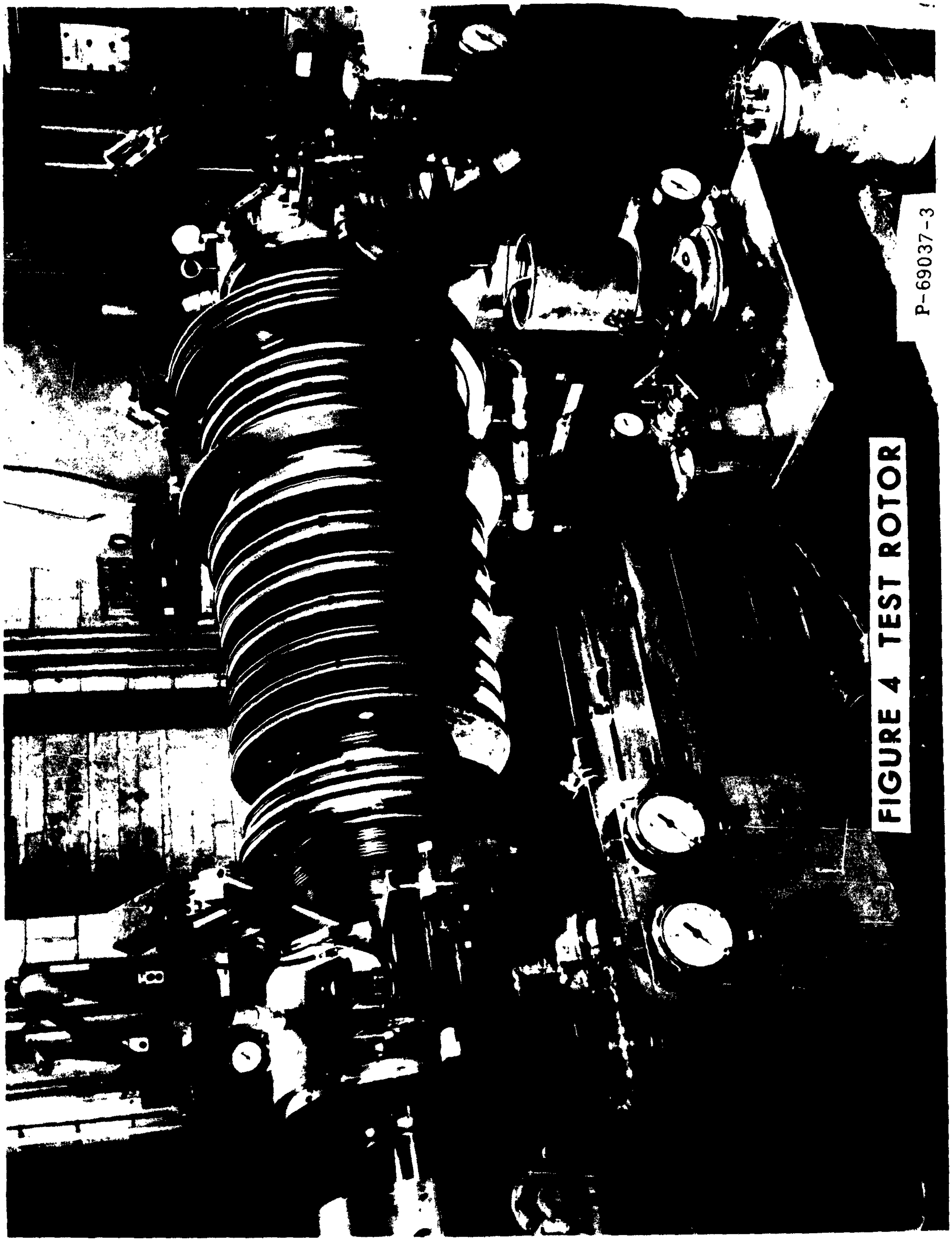


FIGURE 4 TEST ROTOR

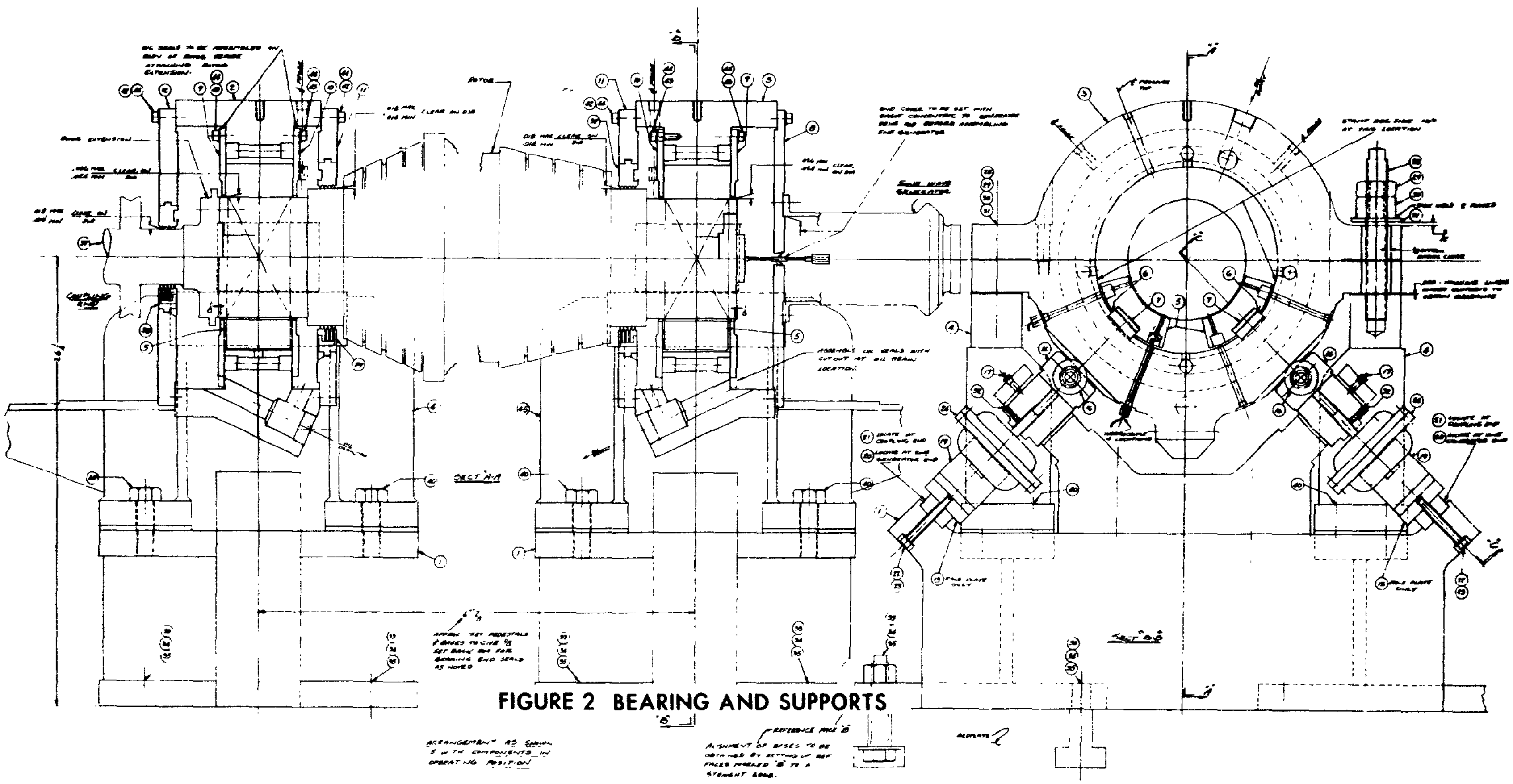


FIGURE 2 BEARING AND SUPPORTS

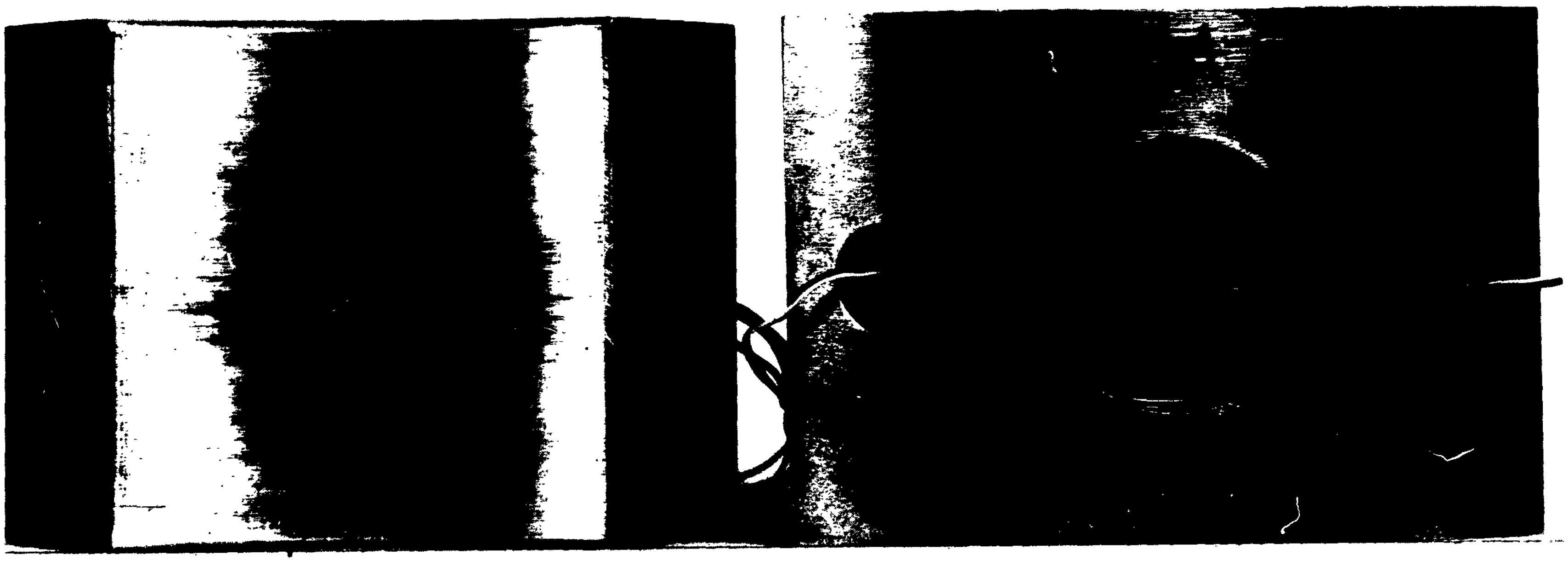
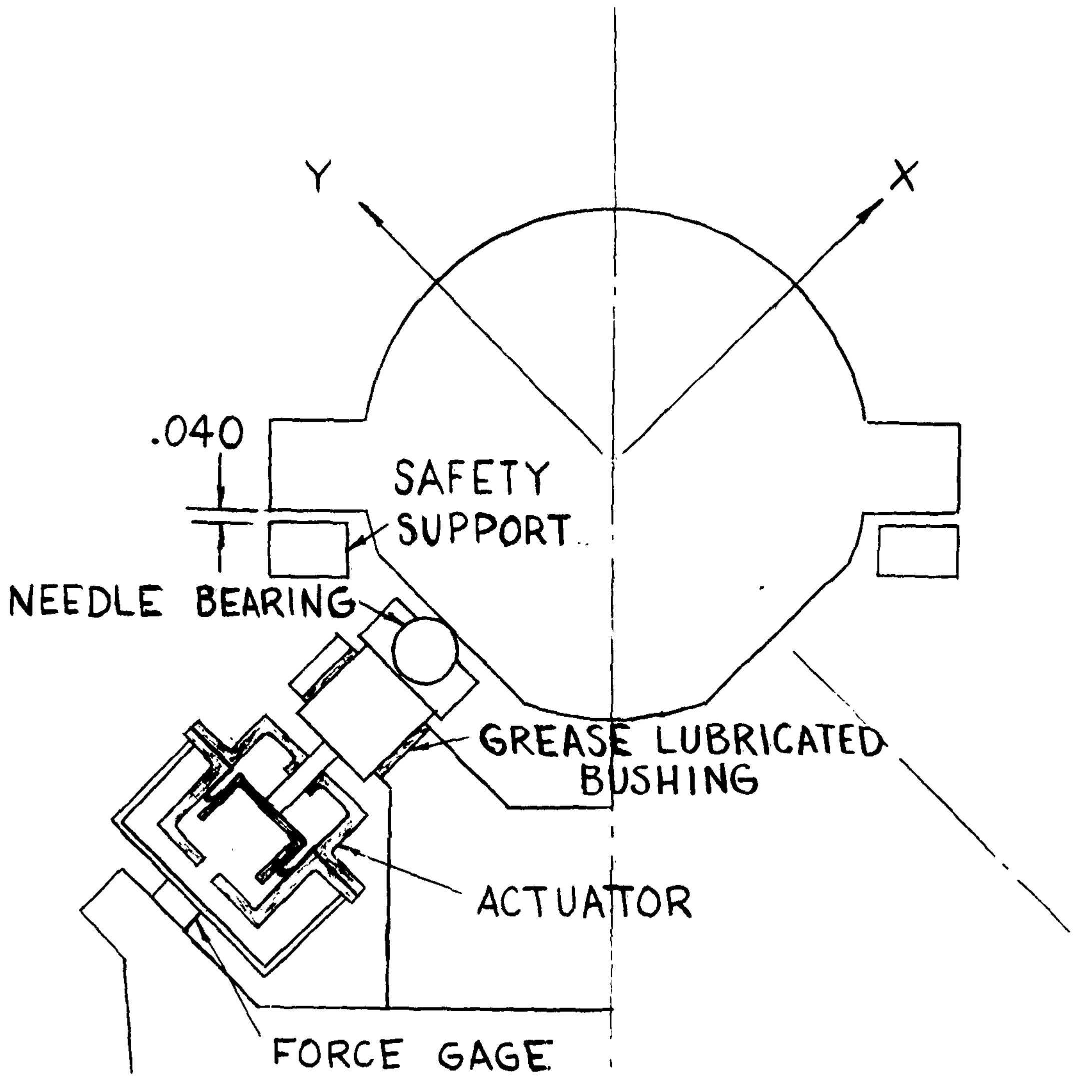


FIGURE 6 BEARING PADS

P-69297

BEARING
FIGURE 7



HYDRAULIC SUPPORT
(LOOKING FROM FREE END)
Figure 8

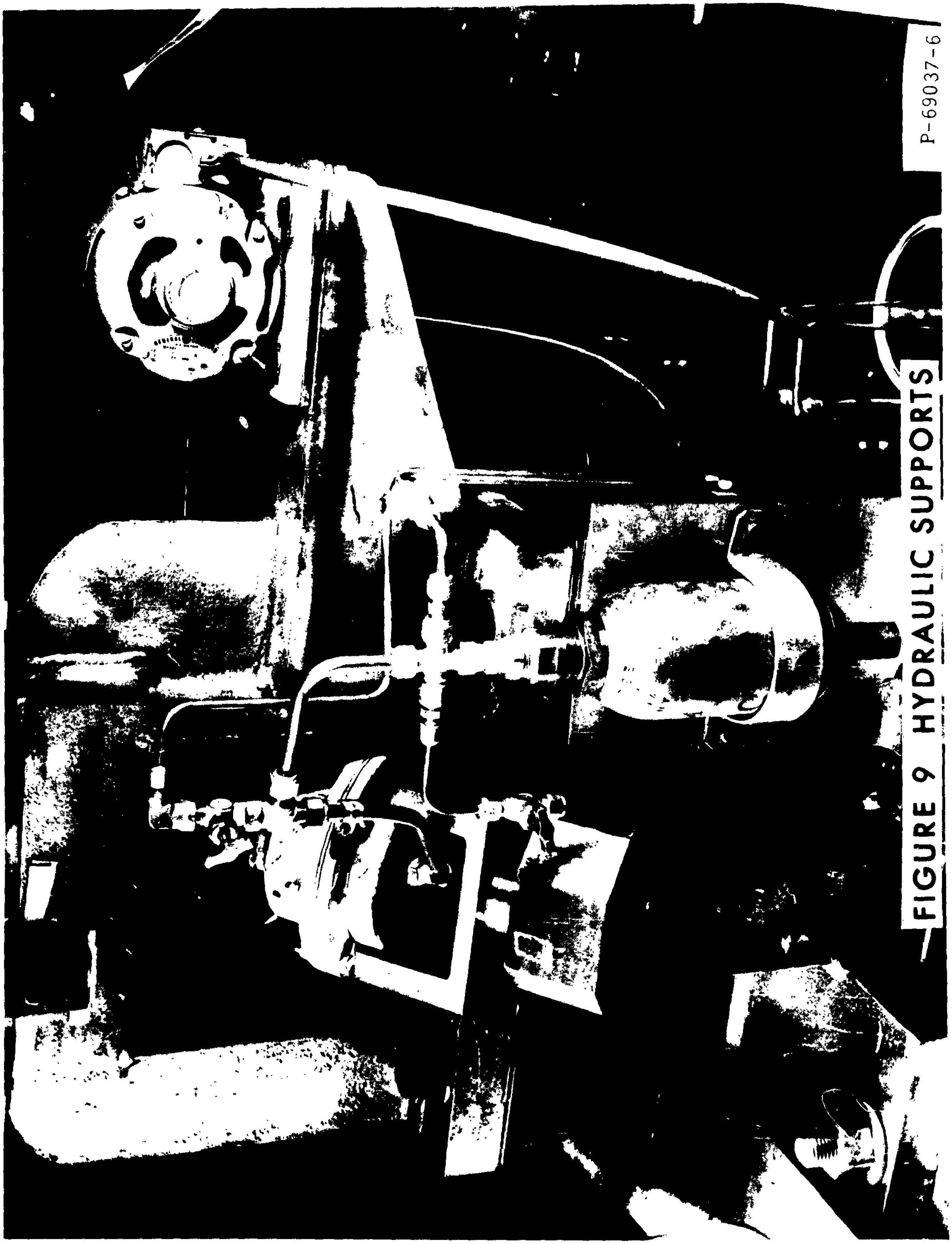


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FIGURE 9 HYDRAULIC SUPPORTS

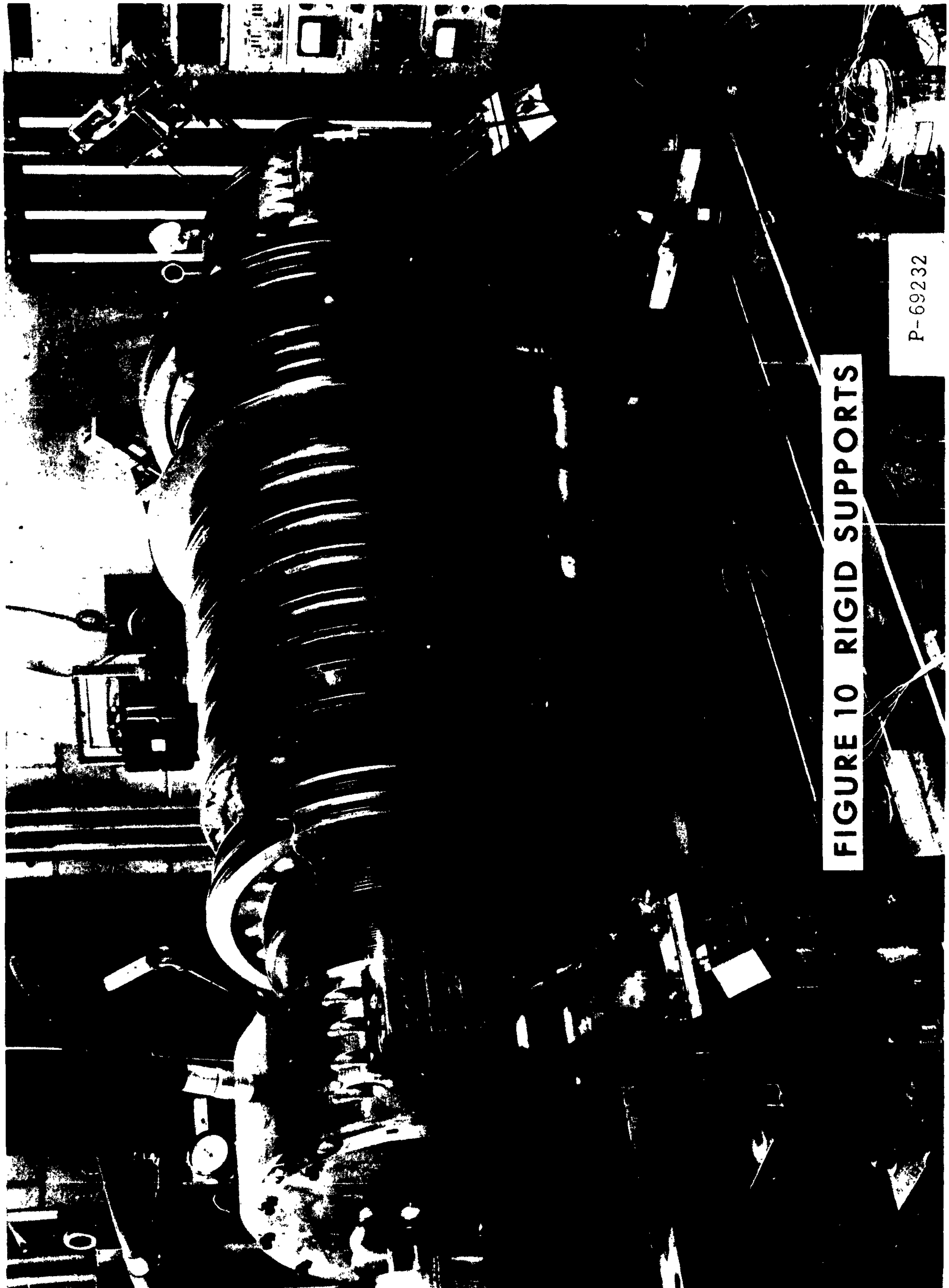


FIGURE 10 RIGID SUPPORTS

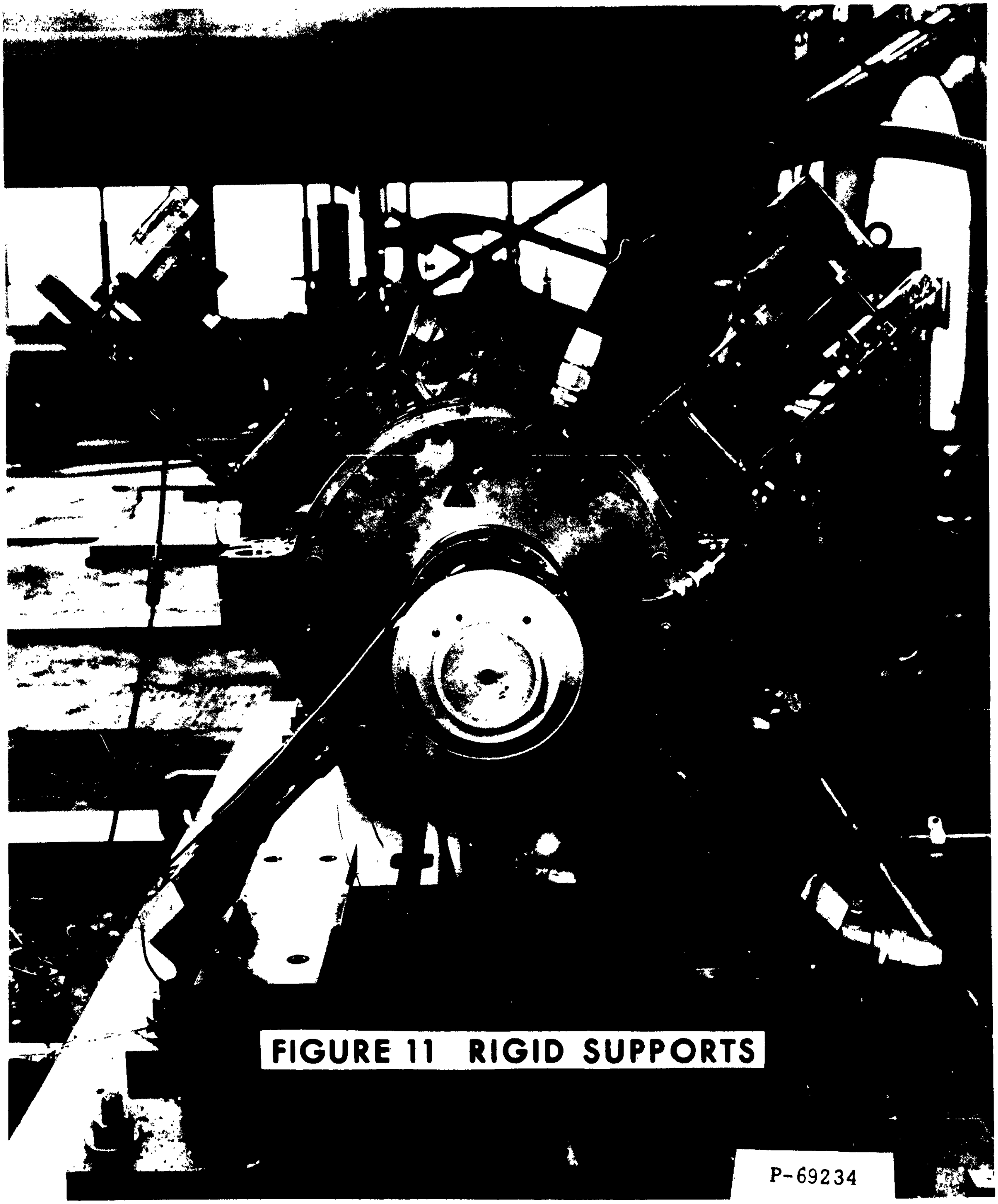


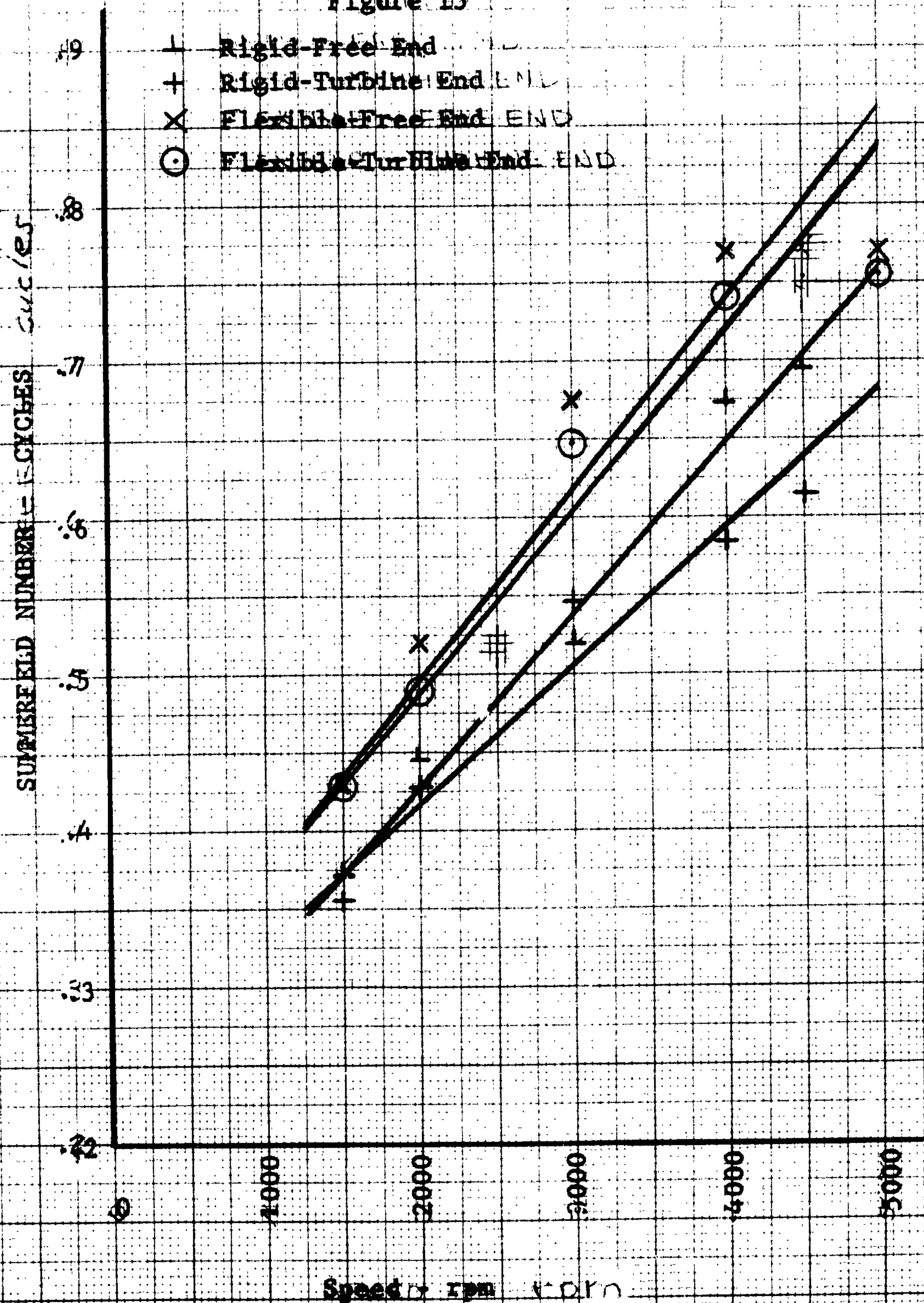
FIGURE 11 RIGID SUPPORTS

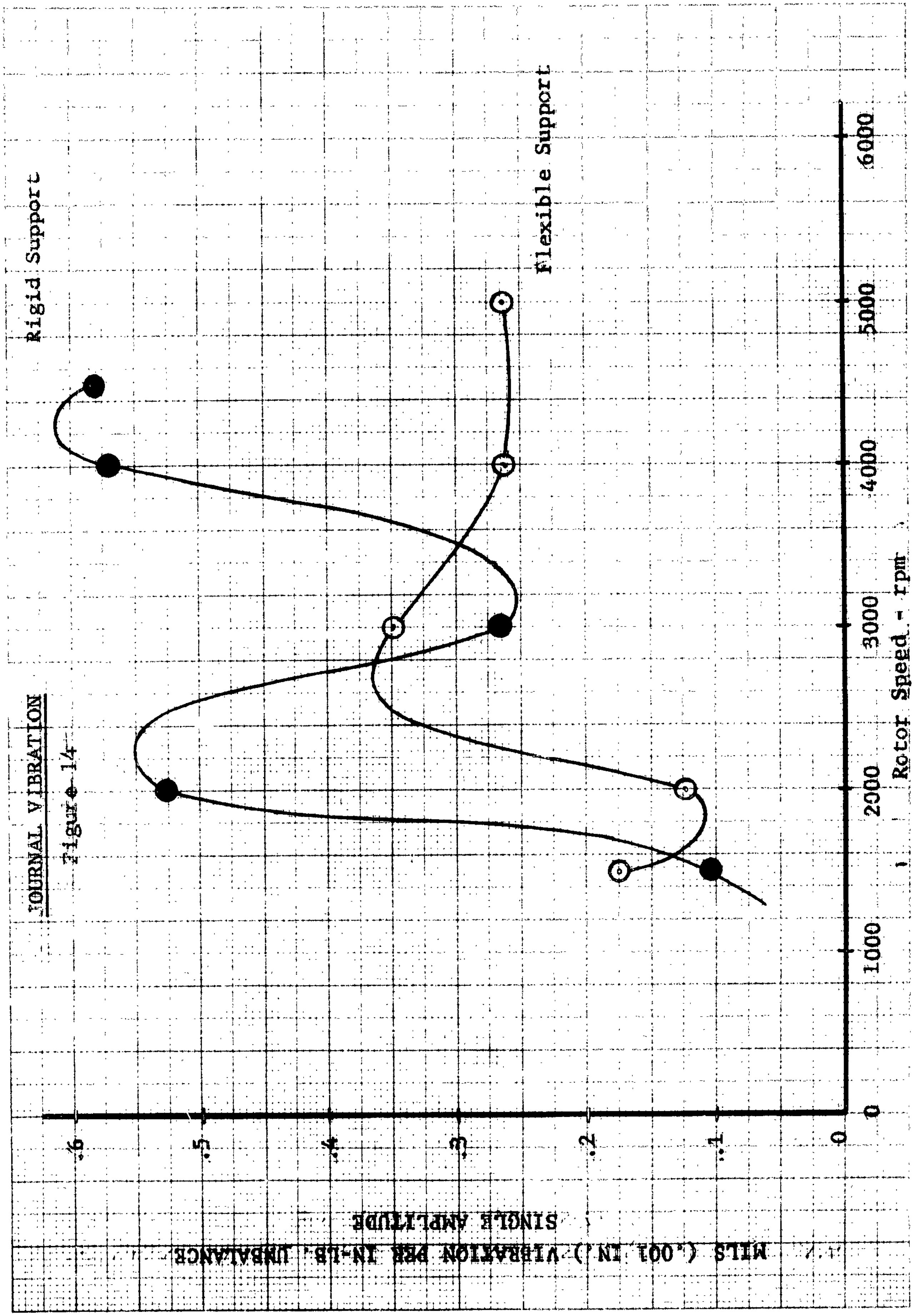
P-69037-2

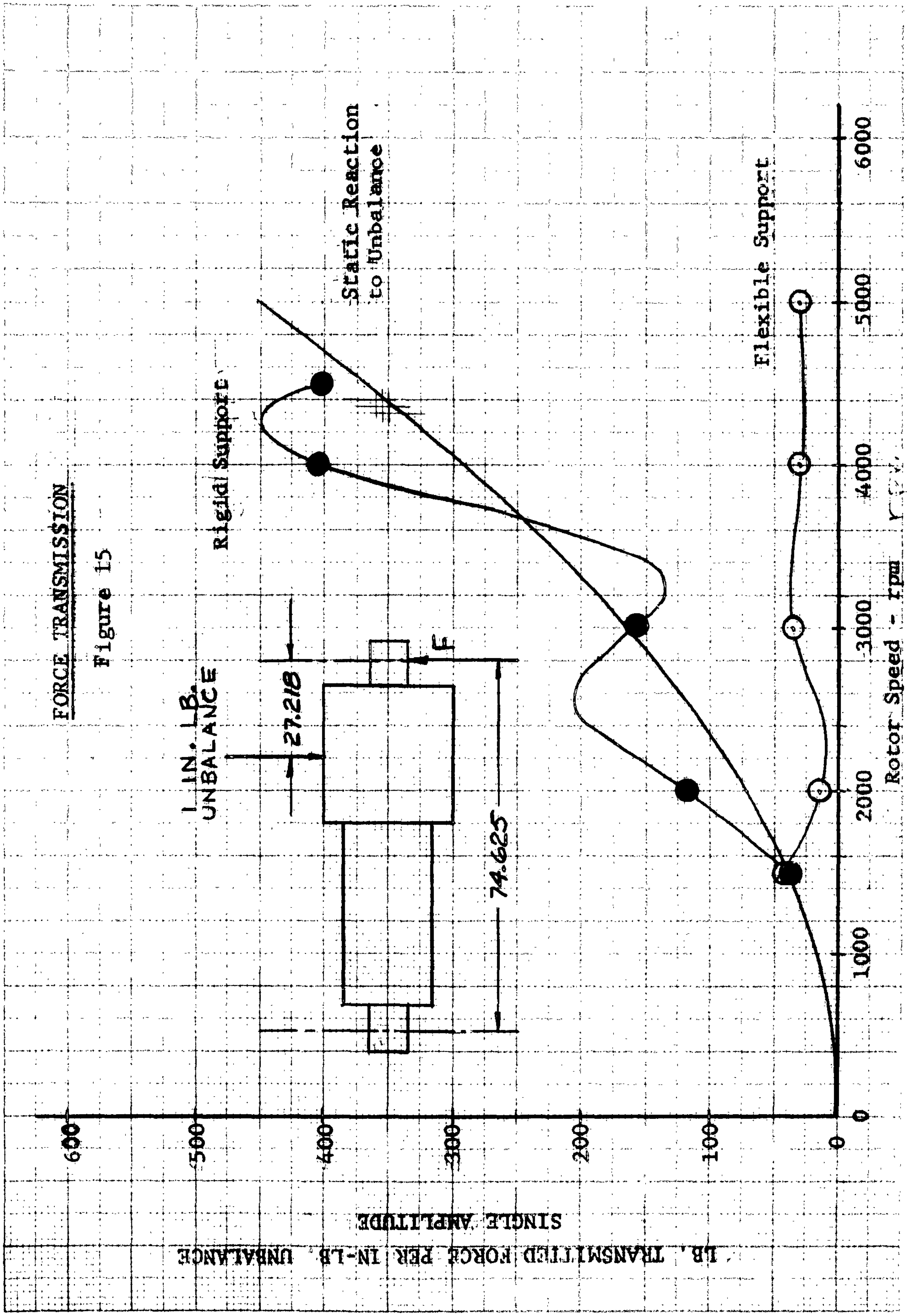
FIGURE 12 INSTRUMENTATION

SOMMERFELD NUMBER
VS
SPEED

Figure 13




JOURNAL VIBRATION
Figure 14
Rigid Support
Flexible Support
Rotor Speed - rpm
0
1000
2000
3000
4000
5000
6000
0
.1
.2
.3
.4
.5
.6
MILS (.001 IN.) VIBRATION PER IN-LB. UNBALANCE
SINGLE AMPLITUDE

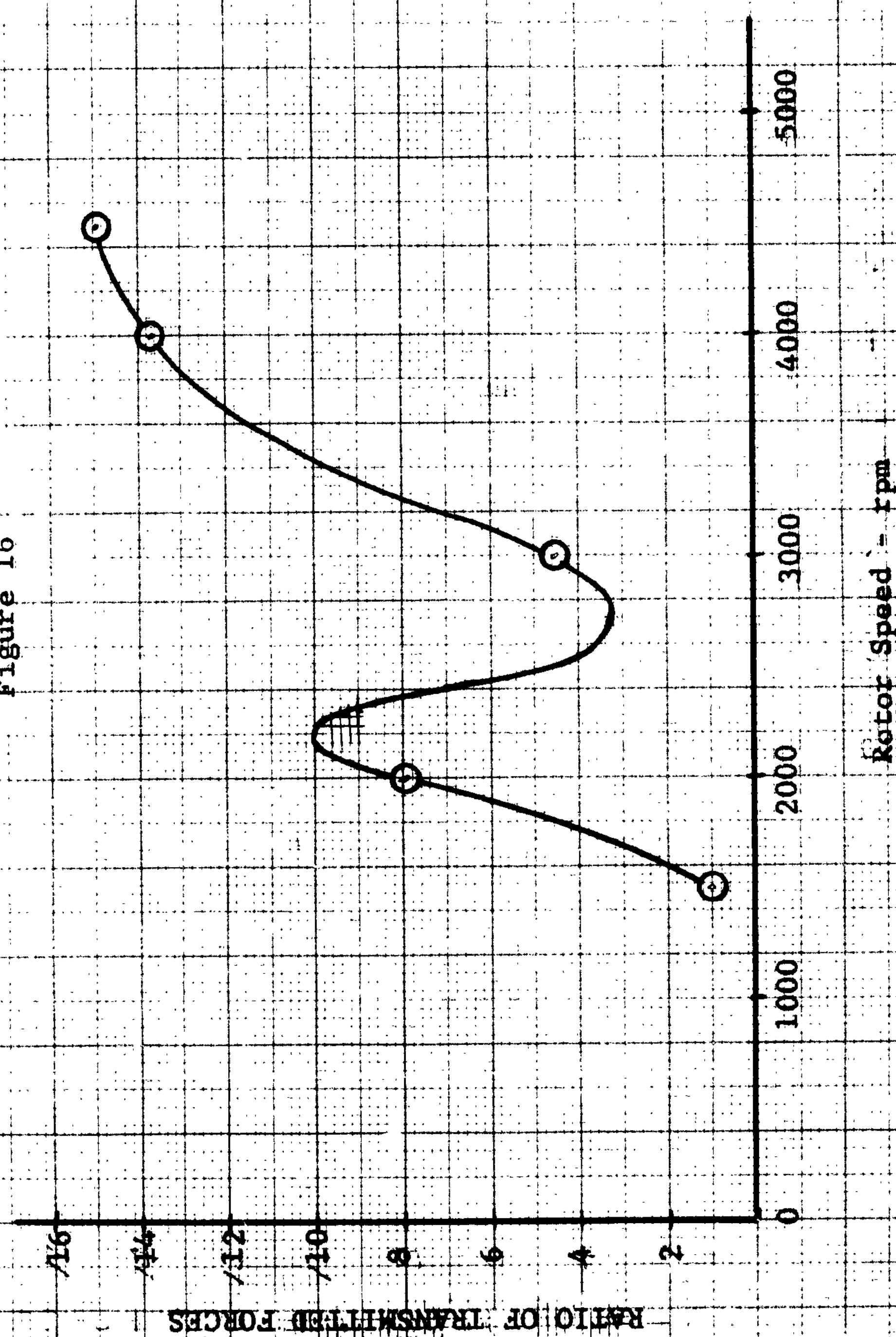


FORM 33001 A

SIGNATURE R.J. Thoman DATE 9/25/64 CURVE NO. ______

RATIO OF TRANSMITTED FORCES

Figure 16



FORM 33001 A

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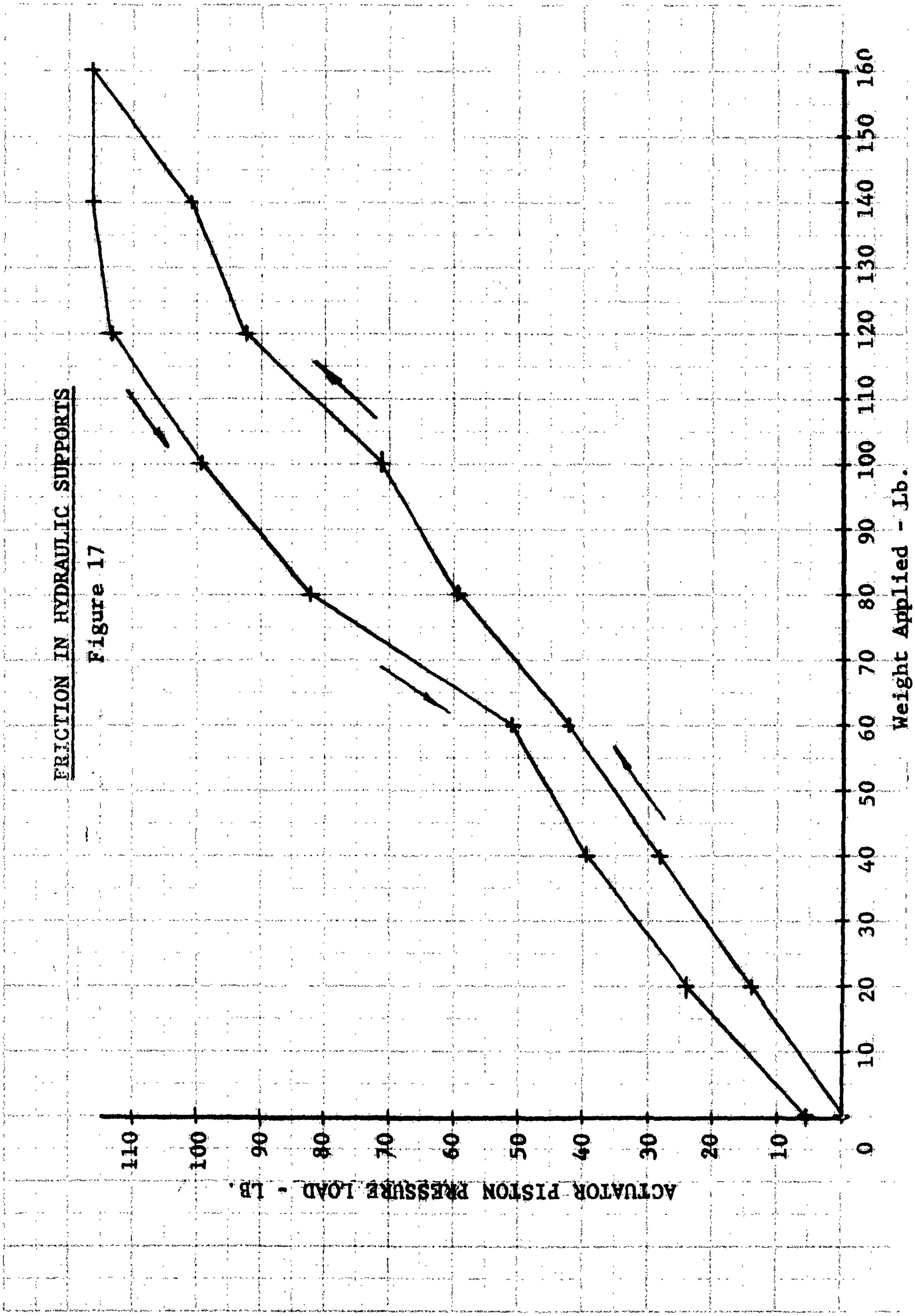

FRICTION IN HYDRAULIC SUPPORTS

Figure 17

FORM 33001 A

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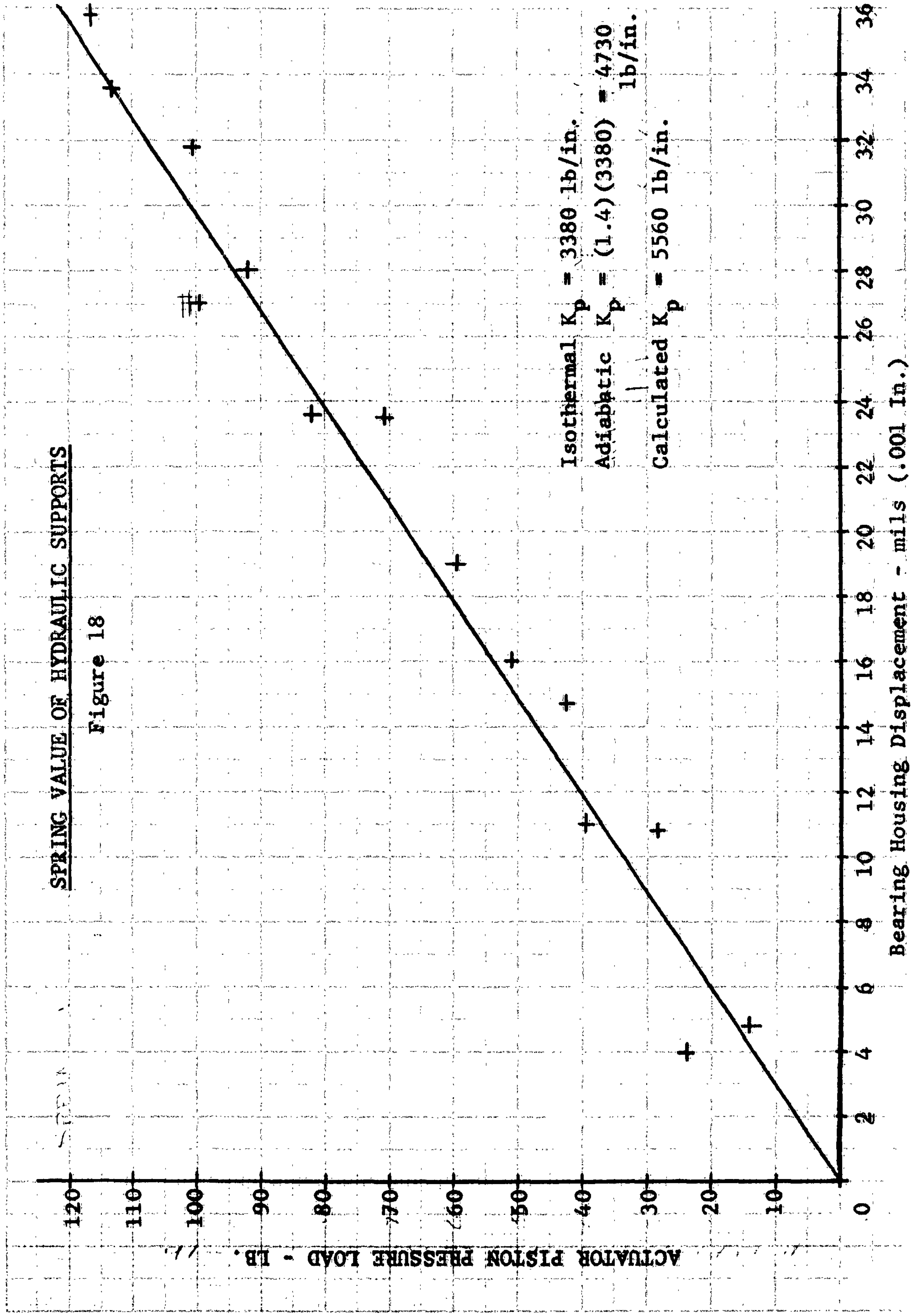

SPRING VALUE OF HYDRAULIC SUPPORTS

Figure 18

FORM 33001 A

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VISCOSITY VS TEMPERATURE

Figure 19

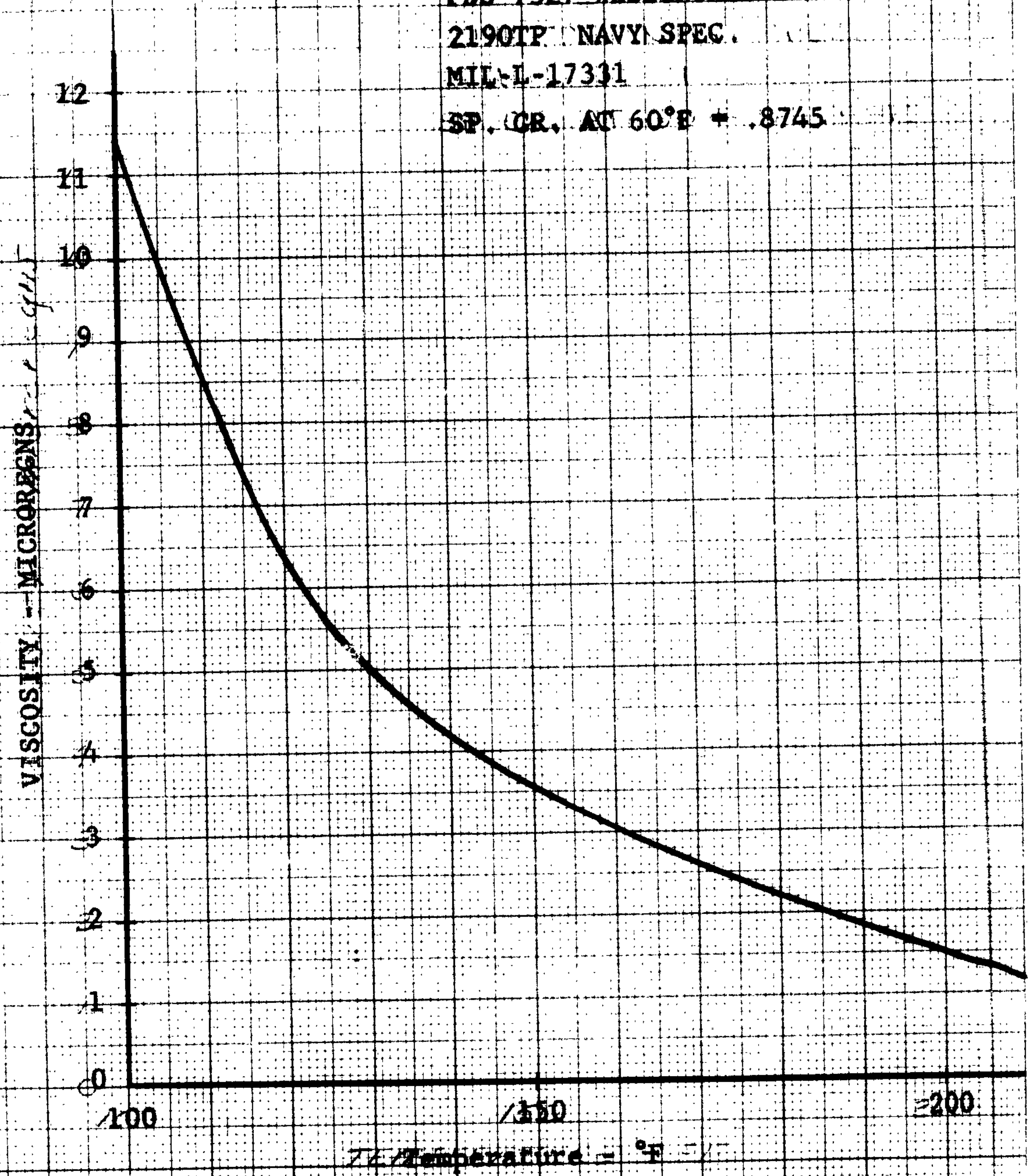
LIGHT TURBINE OIL

PDS 7327 WESTINGHOUSE SPEC.

2190TP NAVY SPEC.

MIL-L-17331

SP. GR. AT 60°F = .8745



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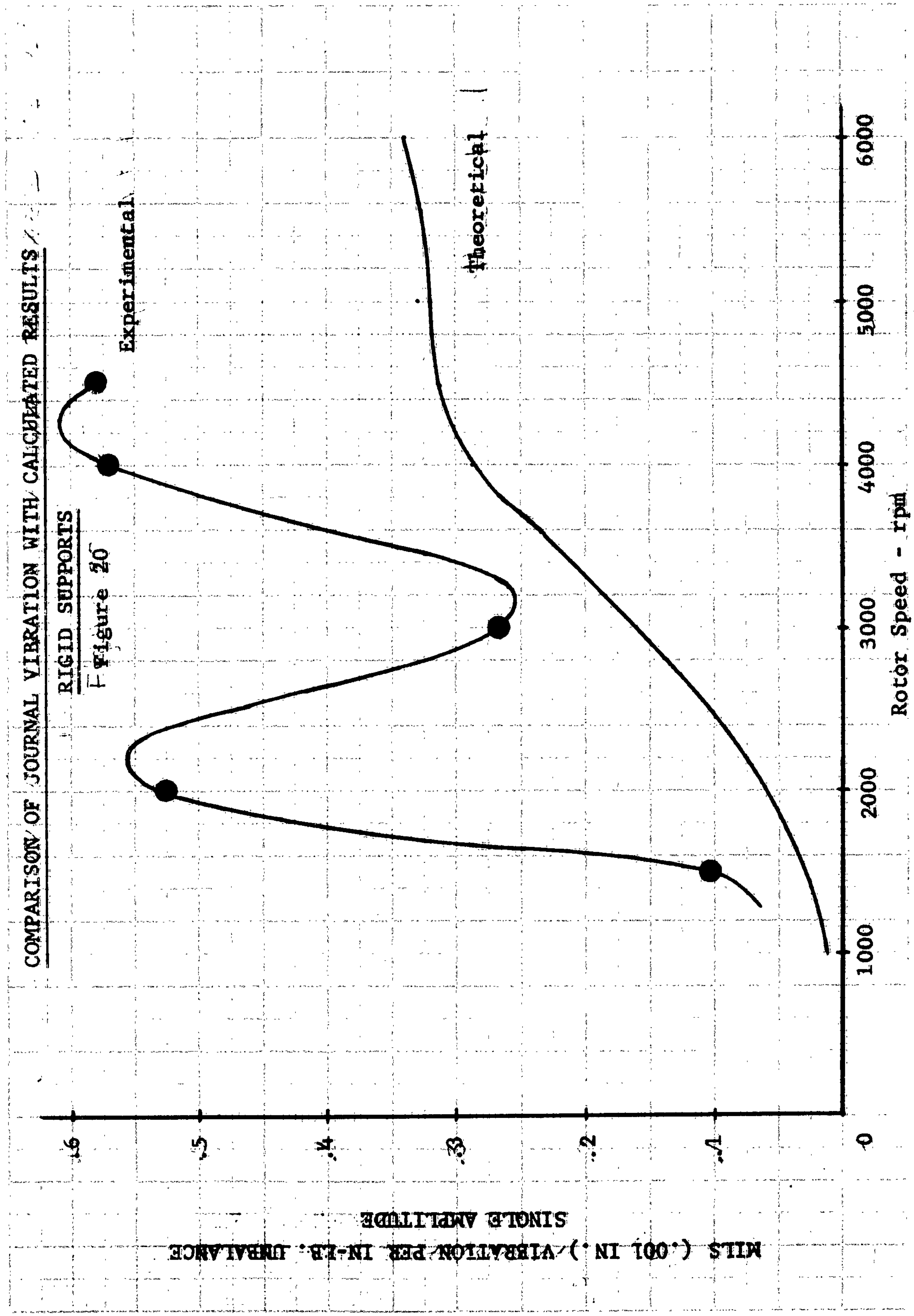
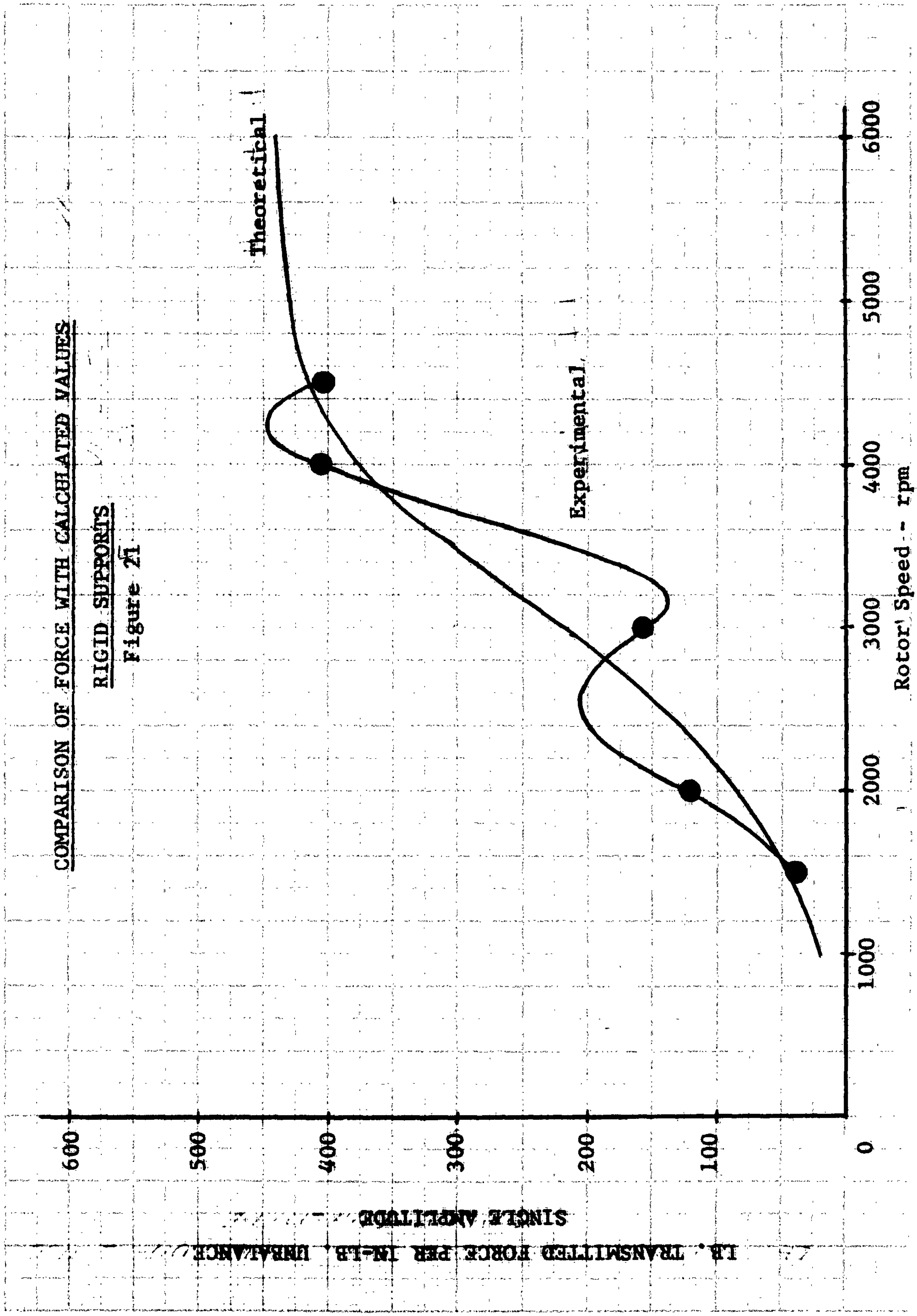


FORM 33001 A

SIGNATURE R.J. Thoman DATE 9/29/64 CURVE NO. ________________



COMPARISON OF FORCE WITH CALCULATED VALUES

RIGID SUPPORTS

Figure 21

FORM 33001 A

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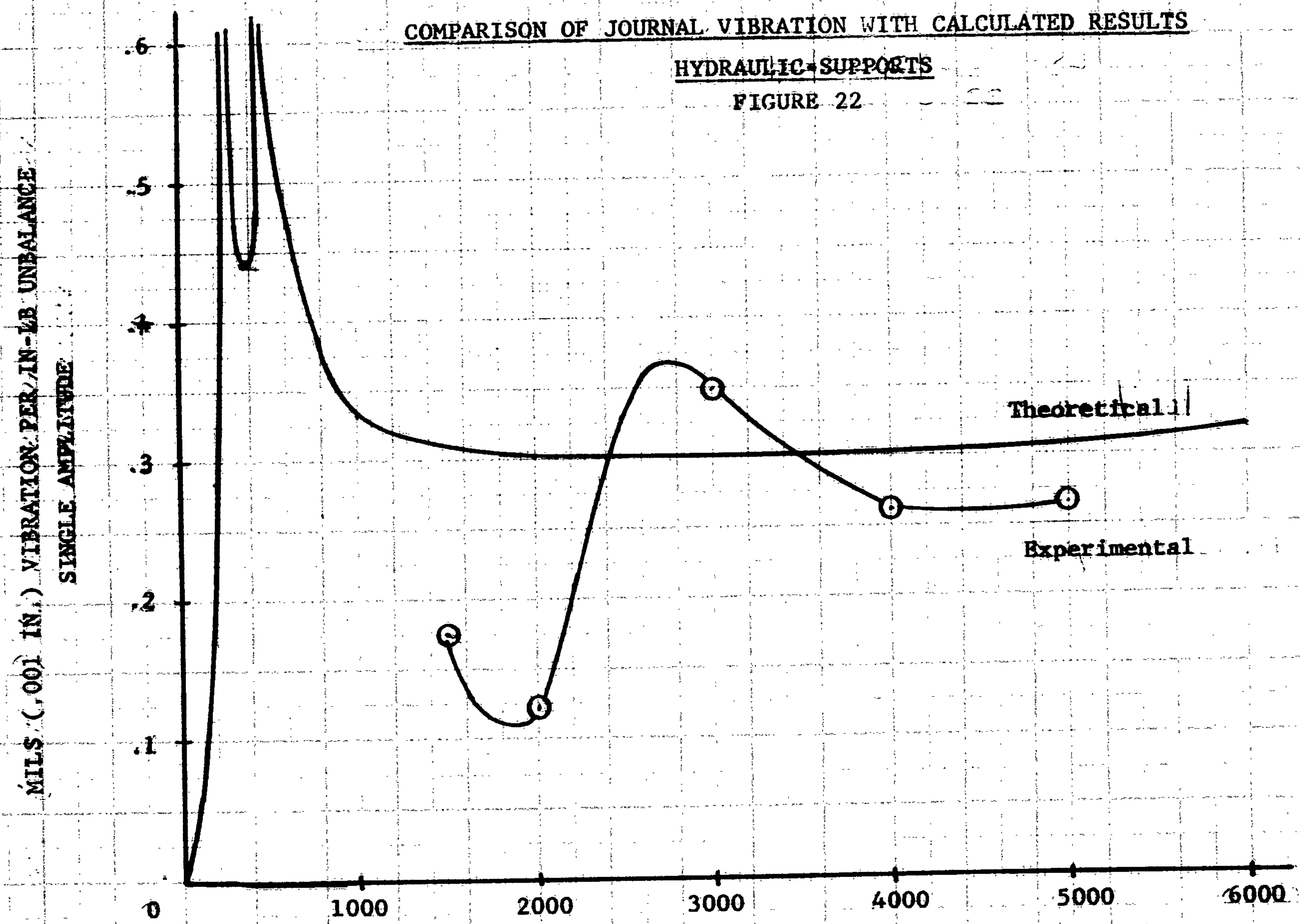
CURVE NO.

COMPARISON OF JOURNAL VIBRATION WITH CALCULATED RESULTS

HYDRAULIC SUPPORTS

FIGURE 22



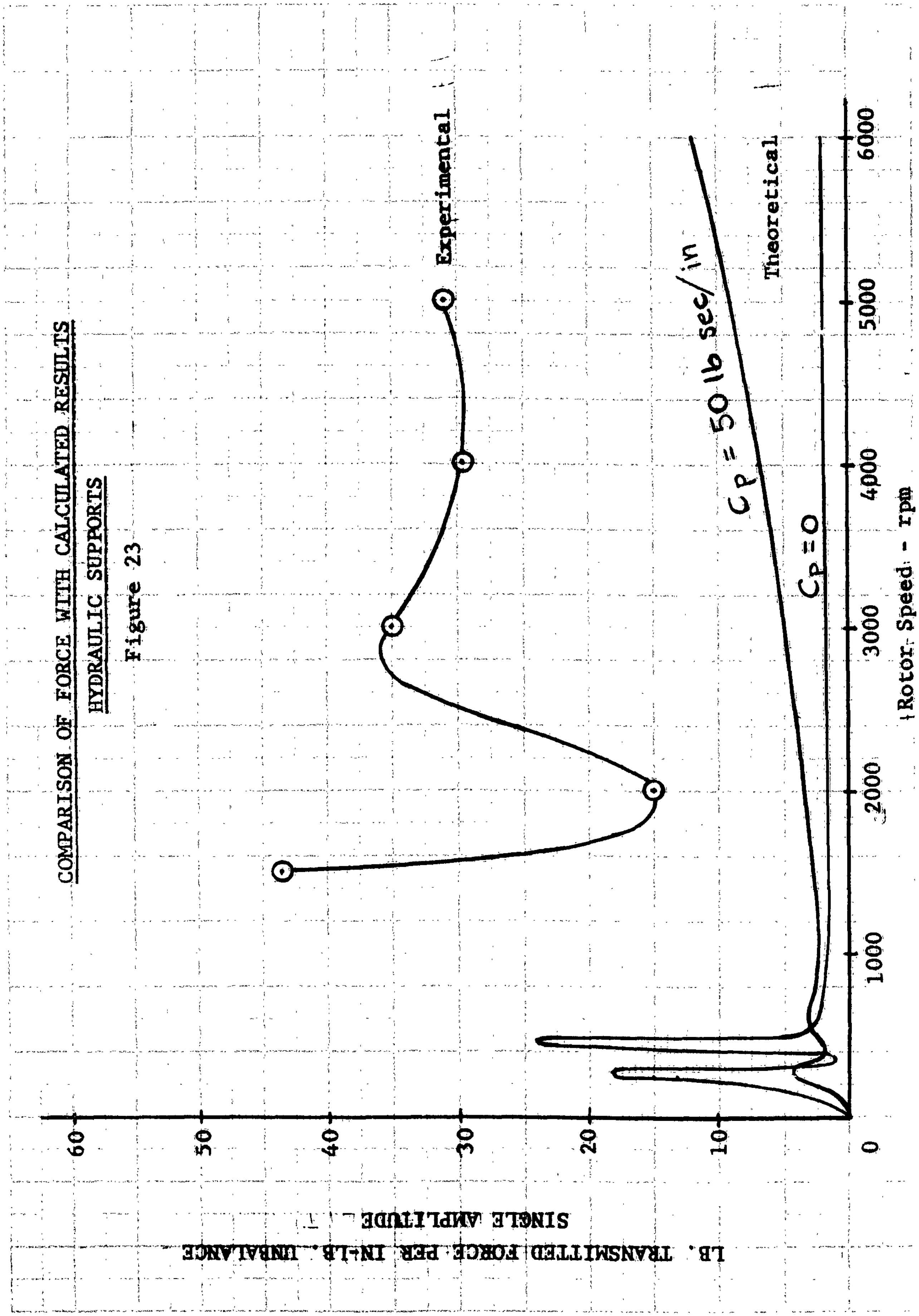
FORM 33001 A

SIGNATURE R. Thoman DATE 9/29/64 CURVE NO.

COMPARISON OF FORCE WITH CALCULATED RESULTS

HYDRAULIC SUPPORTS

Figure 23



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NOMENCLATURE

a, b, c, d	force components
g	gravitational constant
k	adiabatic gas constant
p	pressure
r	balance plane radius
t	time
w	balance plug weight
x	displacement
A	actuator piston area
A, B	angles
C	radial bearing clearance
$\overline{C}$	bearing damping coefficient
$\overline{C}_p$	pedestal or support damping
D	bearing diameter
F	transmitted force
	piston force
$\overline{K}$	bearing spring coefficient
K_p	pedestal or support spring constant
L	bearing axial length
N	speed
P	bearing mean projected area pressure
R	bearing reaction
S	$(\frac{R}{C})^2 \frac{\mu N}{P}$; Sommerfeld No., cycles
T	temperature
V	volume
X, Y	coordinates
ω	angular velocity
μ	viscosity
ϕ	phase angle